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TEXTILE TECHNOLOGY

Obtaining Cotton Fiber Length Distributions from the Beard Test Method Part 1 - Theoretical Distributions Related to the Beard Method

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ABSTRACT

By scanning a fiber beard and generating a fibrogram, certain cotton fiber length parameters can be obtained rapidly. This is the length measurement method used by the High Volume Instrument (HVI™). The objective of this study is to infer fiber length distribution from beard test data. Part 1 of this study deals with the mathematical functions describing length distributions related to the beard method. Eight cottons with a wide range of fiber length were selected and tested on the Advanced Fiber Information System (AFIS™). The measured fiber length data are used for finding the underlying theoretical length distributions. Fiber length distributions by number and by weight are discussed separately, and in both cases a mix of two Weibull distributions shows a good fit to the data. Kolmogorov-Smirnov goodness-of-fit tests were conducted to confirm the findings. Various length parameters such as Mean Length (ML) and Upper Half Mean Length (UHML) are compared between the original distribution from the experimental data and the fitted distributions. A subsequent paper will discuss the inference of fiber length distributions from the beard testing method.

Fiber length is considered the most important property of cotton in marketing and yarn processing. In past decades, the cotton industry and researchers have strived to develop efficient methods to measure the length parameters of cotton fiber. These parameters include Mean Length (ML), Upper Half Means Length (UHML), Short Fiber Content (SFC), and Uniformity Index (UI). Measuring a fiber beard instead of individual fibers provides a rapid account for those fiber length parameters, for example, the widely used High Volume Instrument (HVI™) system (Suh and Sasser 1996). In HVI testing, the specimen fibers are picked up by the needles of a comb/clamp through holes of the HVI Fibrosampler. The collected specimen fibers are in the form of a tapered beard. The beard is brushed and combed to remove loose fibers and fiber crimp. By scanning light attenuation at each length (from the tip of the longest fiber in the beard to the baseline of the clamp), the instrument determines the fiber mass at each length of the beard. The mass-length curve obtained from measuring this tapered beard is called a fibrogram. The original theory of the fibrogram as developed by Hertel (Hertel 1936, 1940) has served as the basis of subsequent cotton length measurement methods based on such tapered fiber beards.

Following Hertel's pioneering work, various developments have been made. Krowicki *et al.* generated fiber length distributions in discrete form from cotton fiber fibrograms. Those generated distributions were presented as graphical bar charts, not as mathematic functions (Krowicki *et al.* 1996). Woo provided a comprehensive appraisal and developed a series of equations for computing different fiber length parameters from fibrograms (Woo, 1967).

Early investigations also included Prier and Sasser's discussion on three different theoretical fiber length distribution density functions: a uniform density and two triangular densities. They claimed that a triangular density could be used to describe short fiber lengths, another triangular density could be used for long fiber lengths, and the uniform density could be used for middle fiber lengths. They further stated that a mixture of these three densities could closely match any set of measured data. However, they did not provide a method to mix the densities. Instead, they concluded that it was not feasible to obtain an explicit expression for the probability density function of the whole fiber length (Prier and Sasser 1971).

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Zeidman *et al.* discussed the range and shape of experimental length distributions and their relationships to length parameters. They concluded that one single parameter could not sufficiently characterize the entire fiber length distribution. More statistical measures, such as mode, dispersion, and shape are needed for distribution (Zeidman *et al.* 1991).

Krifa developed data that defined the modality of fiber length distribution and relationships between modality and other cotton properties such as maturity and strength (Krifa 2006). In later reports, a mixed Weibull distribution was used to describe cotton fiber length and parameters of the mixed Weibull function were discussed regarding the changing of modality during processes (Krifa 2007, 2008). Robert (2005) studied fiber breakage and its relation to length distribution, short fiber, and uniformity. Other efforts focused on: estimation of statistics of fiber length distribution by number and by weight (Cui *et al.* 1998); the impact of beard sampling method on the fiber length distribution and the fibrogram from HVI (Chu and Riley 1997, Cai *et al.* 2009).

The goal of this paper is to study fiber length distributions of fiber beards obtained from the HVI fibrosampler. The relationship between these distributions and the true fiber length distributions will be investigated. The true fiber length distribution will be determined with AFIS. The ultimate objective of this research is to infer fiber length distribution from testing a fiber beard instead of testing individual fibers which is much slower and more expensive. If the distribution functions are known, then all the length parameters can be calculated from it. Since every bale of U.S. cotton is classed by HVI, which employs the beard method, the availability of length distribution from the beard test method will expand the utilization of the classing results. In addition, it will help in understanding the difference of measurement results between HVI and AFIS, which have been reported by earlier research (Cui 1997). To achieve the objective, mathematical functions need to be established that describe the underlying population distributions of the fiber lengths related to HVI measurements.

Determining the length distribution or a mixture of distributions requires quite a number of parameters, and the parameters could be in nonlinear forms, which makes the estimation of the distribution and matching difficult. This paper is part of a series of papers; it focuses on finding and validating the distribution functions dealing with three different types of fiber length distribution that are related to the beard testing method.

MATERIALS AND METHODS

Eight cottons with different length characteristics were selected for this study. The mean lengths by number of these cottons ranged from 1.65 cm to 2.29 cm. Four types of length distributions that are related to beard testing were studied: 1) the length distribution of the original sample, 2) the length distribution of fibers sampled by the HVI Fibrosampler clamp, 3) the length distribution of fibers projecting from the clamp, which is the portion that is actually “seen” and measured by the instrument using a beard method, and 4) the length distribution of the hidden portion of fibers held in the clamp (invisible for an instrument to measure using a beard method). The samples for obtaining the original length distribution were randomly selected by hand in small pinches from the sample population. The HVI Fibrosampler was used to prepare samples for other length distributions. Beards were made by use of the FibroSampler, combed, and brushed to remove loose fibers as is done in HVI length testing with strength testing disabled. The fibers were taken off the HVI clamp for later AFIS testing to determine length distributions of the fibers sampled by the clamp. More beards were prepared in the same way, but the projecting fibers were cut off along the baseline of the HVI clamp. The projecting portion and the hidden portion were collected separately. Figure 1 shows an HVI beard, the projecting fibers spray-dyed to show the hidden portion. Up to 50 beards were needed to collect enough specimen fibers for AFIS testing for each distribution. The collected fibers were gently and thoroughly opened to form thin fiber slivers which were tested on AFIS. For each distribution of each sample, data of at least 35,000 individual fibers was taken. The measurement results (frequency-length relationship) of the above four types of lengths were used to construct the probability density functions (PDF) for fiber length distributions by number and by weight. In this paper the first three types of length distributions are discussed. Their inferences will be discussed in a subsequent paper.

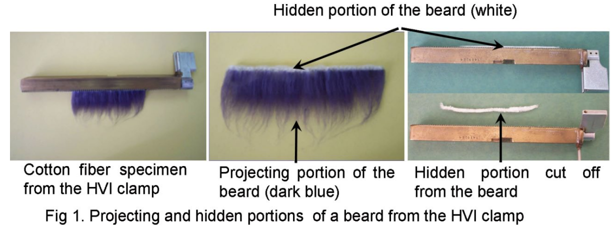

Figure 1. Projecting and hidden portions of a beard from the HVI clamp

Non-linear regression models were constructed with different theoretical distributions such as a normal distribution, and then the Gauss-Newton algorithm and least squares principle were used to solve the models and search for the PDFs that match the PDFs of the AFIS measured data. The statistical software SAS® was used for computations. Computation results showed that a mixture of two two-parameter Weibull distributions fits the data very well. That is, each sample can be characterized by a mixture of Weibull distributions. Each mixed distribution has five parameters, which are determined directly using SAS.

RESULTS AND DISCUSSION

Theoretical Distribution of Fiber Length. Let $X_1, X_2, \ldots, X_n$ denote a random sample from a population with a cumulative distribution function (CDF) $F(x)$, and let $F_n(x)$ = (the number of $X_i \leq x/n$) denote the empirical cumulative distribution function (ECDF) of the random sample. The ECDF is the proportion of observations less than or equal to x. It has been shown that the ECDF is the non-parametric maximum likelihood estimate of F: when the sample size n goes to infinity, the ECDF converges to the true CDF F (Schorack and Wellner, 1986). That is, the probability of $\lim_{n\to\infty} F_n(x) = F(x)$ equals one. In other words, since the sample sizes of the data sets used in this paper are considerably large (>30,000), the ECDFs based upon these data sets can be considered as the underlying population CDFs of corresponding data sets. These ECDFs were the targets of fitting of the distribution functions sought.

The least squares principle was used to determine the models. The probability density function (PDF) $g^*(x)$ is a least squares (LS) estimate of PDF $f^*(x)$ if $\int_0^\infty [g(x) - f^*(x)]^2\,dx$ is minimized at $g^*(x)$ among all choices of $g(x)$. To find a distribution to fit the cotton length data, first divide the interval [0, 3] (inch) into 30 subintervals of equal length. The interval [0, 3] is selected because it practically covers the entire possible length of cotton fibers: 0 to 3 inches (0 to 7.62 cm). Then count the frequencies $h(x)$, i.e., the number of fibers with length falling into a subinterval. That is, $h(x)$ equals the number of fibers with length falling into the subinterval covering x. This $h(x)$ is the PDF of the cotton fiber length by number and is used as the underlying PDF of the experiment data. It is actually the $f^*(x)$ in the LS estimation process mentioned above. To find the LS estimate $g^*(x)$ of $h(x)$, various distributions were tried including normal, lognormal, beta, Weibull, mixture of normal distributions, etc., and it was determined that a mixture of two Weibull distributions can fit the data very well. The PDF of a two-parameter Weibull distribution is given by

$$f(x;\lambda,\theta) = \lambda\theta x^{\lambda-1} e^{-\theta x^{\lambda}},\ x>0,\ \lambda>0,\ \theta>0 \qquad (1)$$

where λ is the shape parameter and θ is the scale parameter. The CDF is given by

$$F(x;\lambda,\theta) = \int_0^x f(t;\lambda,\theta)\,dt = 1 - e^{-\theta x^{\lambda}},\ x>0 \qquad (2)$$

The PDF of a mixture of two Weibull PDFs is given by

$$f(x;\alpha,\lambda_1,\theta_1,\lambda_2,\theta_2) = \alpha f_1(x;\lambda_1,\theta_1) + (1-\alpha) f_2(x;\lambda_2,\theta_2) \qquad (3)$$

where $0<a<1$ and $f_i(x;\lambda_i,\theta_i)$ is the PDF of a Weibull distribution, i = 1, 2. So, the PDF of the mixture of two Weibull PDFs contains five parameters: a, λ_1, θ_1, λ_2, θ_2. Similarly, the CDF of the mixture is $F(x;\alpha,\lambda_1,\theta_1,\lambda_2,\theta_2) = \alpha F_1(x;\lambda_1,\theta_1) + (1-\alpha)F_2(x;\lambda_2,\theta_2)$, where $F_i(x;\lambda_i,\theta_i)$ is the CDF of $f_i(x;\lambda_i,\theta_i)$, i = 1, 2. The value of α determines the amount of contribution of a Weibull distribution to the mixed distribution. For instance, if α is close to unity, then PDF1, $f_1(x;\lambda_1,\theta_1)$, has a significantly larger contribution to the mixed distribution than does PDF2, $f_2(x;\lambda_2,\theta_2)$. To simplify notations, we use $f(x)$ for $f(x;\alpha,\lambda_1,\theta_1,\lambda_2,\theta_2)$ and $F(x)$ for $F(x;\alpha,\lambda_1,\theta_1,\lambda_2,\theta_2)$. As mentioned earlier, due to the large sample size, this empirical PDF $h(x)$ can approximate the underlying population PDF of the data. Therefore, the LS estimate $g^*(x) = f(x)$, mixture of Weibull distributions, can be used as the population PDF by number. Once the functional form of the population PDF is obtained, various cotton length parameters can be computed.

The Kolmogorov-Smirnov goodness-of-fit test was performed to verify that the mixture of two Weibull distributions fits the data. This test can be explained as the following. Let the hypothesis be "the data follows distribution $G(x)$", where $G(x)$ is a completely specified CDF. Let $F_n(x)$ denote the ECDF of a data. Define

$$D_n = \sqrt{n} \sup_{-\infty<x<\infty} \left|F_n(x) - G(x)\right| \qquad (4)$$

It is shown (Mood, *et al.* 1974) that when the hypothesis is true and the sample size n is large, D_n is approximately distributed as $D(x)$, where

$$D(x) = 1 - 2\sum_{i=1}^{\infty}(-1)^{i-1}e^{-2i^2x^2},\ \ x>0 \quad (5)$$

It is clear that if the hypothesis is false, then D_n tends to be large; hence for a given significance level α, one would reject the hypothesis if D_n is greater than the critical value d_α, where d_α is determined so that $D(d_\alpha)$=1- α. For example, when α =0.10, $d_\alpha \approx$ 1.22, and when α =0.05, $d_\alpha \approx$ 1.44. In our computations, the Kolmogorov-Smirnov goodness-of-fit test is performed in the following steps:

Step1. Since in practice it is common that the sample size is usually around 2000 to 3000, we randomly re-sample n=2500 individual fibers from a data set.

Step2. Fit a mixture of two Weibull distributions to the re-sampled data set. This fitted mixture CDF is $G(x)$, and the ECDF of the re-sampled data is $F_n(x)$ in Equation 4.

Step3. Use α =0.10. Compare D_n with d_α =1.22. If D_n is less than 1.22, the Kolmogorov-Smirnov statistic is not significant, and then the hypothesis that these 2500 re-sampled data points follow a mixture of two Weibull distributions is accepted.

Step4. Repeat Steps 1-3 500 times and record the number of times that the hypothesis is accepted. In all the eight sets of fibers, acceptance was higher than 95% of the 500 tests for each set.

The test was performed for original, comb-sampled, and projecting fibers of all eight cottons, and the hypothesis is accepted in all cases with a p-value greater than 0.10. Therefore, it can be concluded that the fiber length distribution by number can be described using a mixture distribution of two Weibull distributions.

The previous discussion concerns the fiber length distribution by number. The shorter fibers may have a large number portion, but not a large weight portion. Therefore, in practice, the cotton fiber length distribution by weight is more commonly used than the distribution by number. Fiber length distribution by weight was then considered. The fiber length has a very weak correlation with the fiber linear density, and an assumption has been widely used that the fiber length and fiber linear density are independent (Zeidman *et al.* 1991, Cui *et al.* 1998). Experimental data also support this assumption. For the eight cotton samples discussed in this paper, the average correlation between individual fiber length and fineness is only 0.155. As previously defined, let $h(x)$ denote the frequency function of a data set. Further, let $\bar{x}$ denote the sample mean of the data set. By using the assumption of independency between fiber length and fiber linear density, the frequency function by weight is given by $h_w(x) = xh(x)/\bar{x}$ (Zeidman *et al.*, 1991). A mixture of two Weibull distributions is used to fit $h_w(x)$. Kolmogorov-Smirnov goodness-of-fit test is also performed for the length by-weight and the same conclusions were obtained as for the length by number.

Table 1 and Table 2 give, respectively, parameters of the estimated mixture distributions by number and by weight for the eight different cottons. (λ_1, θ_1) are the parameters of the Weibull distribution on the left (labeled pdf1 in figures) of the two Weibull distributions in the mixture, which mainly represents shorter fibers, and the one on the right determined by (λ_2, θ_2) represents longer fibers (labeled pdf2 in figures). Figure 2 shows the PDFs (by number) of the four types of length distributions as mentioned in the Materials and Method section of one cotton sample. For each of the eight cottons, graphical comparisons between the PDF of the estimated mixture distributions and the PDF of data are performed. However, for simplicity, only the graphs of the PDF's by number and by weight for Sample ID 34 and 38 (Figures 3-14 in the appendix) are presented. Table 3 presents numerical comparisons of some quality parameters of all eight samples. In addition, the approach described in this section was used to fit another 28 cottons with micronaire ranging from 2.92 to 5.52. The mixture of Weibull distributions also gave satisfactory results.

Fiber Length Parameters from the Mixture of Weibull Distributions. In this section, comparisons between the parameters obtained from the original data and that obtained from the estimated mixture distribution are presented.

The mean and the variance of a Weibull distribution with parameters λ and θ are, respectively, given by

$$\mu = \Gamma(1+1/\lambda)\theta^{-1/\lambda} \quad (6)$$

and

$$\sigma^2 = [\Gamma(1+2/\lambda) - \Gamma^2(1+1/\lambda)]\theta^{-2/\lambda} \quad (7)$$

where $\Gamma(\beta) = \int_0^\infty x^{\beta-1}e^{-x}dx$ denotes the gamma function. Hence the mean and the variance of a mixed Weibull distribution are, respectively,

$$\mu_{mix} = \alpha\mu_1 + (1-\alpha)\mu_2 \quad (8)$$

and

$$\sigma_{mix}^2 = E_{mix}(x^2) - \mu_{mix}^2 = \alpha(\sigma_1^2 + \mu_1^2) + (1-\alpha)(\sigma_2^2 + \mu_2^2) - [\alpha\mu_1 + (1-\alpha)\mu_2]^2 \quad (9)$$

where μ_i and σ_i^2 denote, respectively, the mean and the variance of a Weibull distribution with parameters λ_i and θ_i, i=1, 2.

The mean fiber length can be obtained by $\mu = \int_0^L [1-F(u)]du$, where L is the maximum fiber length in the distribution. The fibrogram is a curve defined by $B(x) = \frac{1}{\mu}\int_x^L [1-F(u)]du$ (Zeidman *et al.*, 1991). The UHML is the mean length of fibers longer than the median value of the weight distribution of fibers. Let M denote the weight median value. Then M is such that $\int_0^M uf(u)du = \frac{\mu}{2}$. It is known that the UHML is the x-intercept of the tangent line to the fibrogram $B(x)$ passing through y-intercept 0.5 and can be obtained by

$$UHML = \frac{\int_M^L uf(u)du}{1-F(M)} \quad (10)$$

The LHML is given by

$$LHML = \frac{\int_0^M uf(u)du}{F(M)} \quad (11)$$

The SFC by number is the number proportion of fibers shorter than 0.5 inches in the cotton. When using the fitted distribution the SFC by length equals simply $\alpha F_1(0.5;\lambda_1,\theta_1) + (1-\alpha)F_2(0.5;\lambda_2,\theta_2)$. The SFC by weight can be defined similarly.

Plugging the fitted mixed Weibull distribution into these formulas, the estimated mean length, UHML, LHML, SFC and CV are given in Table 3. For each type of distribution, there are two rows of values in Table 3: the top row ("Data") is from the experimental data and the second row ("Weibull") is from the fitted distribution. It can be seen from Table 3 that the fiber length parameters obtained from the experimental data and those obtained from the mixed Weibull distribution match very well, especially if we consider the natural non-uniformity of cotton length, the third decimal of the data usually does not have statistical significance. The R^2 between model predicted values and measured values is very high, for example, both the R^2 of Mean Length by number and the R^2 of Short Fiber Content by number are >0.99, indicating an excellent fit of the mixture of the Weibull distributions to the measured data.

Table 1. Estimation of Mixture Distribution Parameters (by Number)

ID	Type	α	λ_1	θ_1	λ_2	θ_2
30	Original	0.229	2.114	1.336	3.481	0.073
	Sampled	0.502	1.980	0.390	4.283	0.032
	Projecting	0.946	2.325	0.465	3.383	5.640
31	Original	0.840	3.667	0.060	2.197	1.879
	Sampled	0.166	2.143	1.204	3.752	0.055
	Projecting	0.039	3.129	4.471	2.367	0.407
33	Original	0.301	1.881	0.910	4.042	0.026
	Sampled	0.714	4.006	0.027	1.958	0.614
	Projecting	0.031	3.055	4.033	2.335	0.342
34	Original	0.508	5.178	.006	1.841	0.338
	Sampled	0.525	5.032	.007	2.076	0.277
	Projecting	0.060	2.960	3.082	2.449	0.292
35	Original	0.515	4.931	.007	1.921	0.332
	Sampled	0.468	4.924	.008	2.014	0.303
	Projecting	0.934	2.358	0.296	2.723	2.149
36	Original	0.554	4.971	.005	1.674	0.416
	Sampled	0.591	4.612	.009	1.995	0.340
	Projecting	0.090	2.370	1.634	2.369	0.280
37	Original	0.623	4.632	.007	1.738	0.491
	Sampled	0.558	4.884	.005	2.034	0.275
	Projecting	0.057	2.527	1.820	2.302	0.276
38	Original	0.645	5.151	.003	1.779	0.502
	Sampled	0.546	5.396	.002	1.942	0.278
	Projecting	0.099	2.245	1.404	2.488	0.206

Table 2. Estimation of Mixture Distribution Parameters (by Weight)

ID	Type	α	λ_1	θ_1	λ_2	θ_2
30	Original	0.531	4.828	0.017	2.356	0.181
	Sampled	0.481	5.017	0.014	2.731	0.124
	Projecting	0.820	2.751	0.224	2.640	0.387
31	Original	0.706	4.782	0.018	2.116	0.196
	Sampled	0.728	4.606	0.021	2.248	0.180
	Projecting	0.721	2.828	0.218	2.899	0.206
33	Original	0.618	5.188	0.006	2.252	0.175
	Sampled	0.666	4.817	0.009	2.532	0.143
	Projecting	0.842	2.863	0.147	3.145	0.261
34	Original	0.607	5.653	0.003	2.373	0.123
	Sampled	0.181	2.748	0.224	5.127	0.006
	Projecting	0.998	2.891	0.152	5.482	5.723
35	Original	0.524	5.846	0.002	2.632	0.096
	Sampled	0.550	2.770	0.087	5.902	0.003
	Projecting	0.998	2.758	0.151	419.980	9.9E-9
36	Original	0.430	2.456	0.104	6.087	0.001
	Sampled	0.512	5.846	0.002	2.835	0.075
	Projecting	0.996	2.776	0.148	8.161	0.480
37	Original	0.386	2.358	0.114	5.682	0.002
	Sampled	0.539	5.960	0.001	2.836	0.070
	Projecting	0.999	2.750	0.137	107.020	145.996
38	Original	0.342	2.295	0.116	6.182	0.001
	Sampled	0.575	6.410	0.001	2.723	0.073
	Projecting	0.998	2.872	0.110	11.964	10E-7

Table 3. Estimation of Some Length Quality Parameters by Number

ID	Type	Source	Mean (cm)	UHML (cm)	LHML (cm)	CV	SFC	UI (%)
30	Original	Data	1.666	2.497	1.250	0.453	0.318	80.42
		Weibull	1.647	2.466	1.237	0.448	0.323	80.25
	Sampled	Data	1.740	2.511	1.331	0.415	0.271	81.23
		Weibull	1.728	2.502	1.322	0.425	0.276	81.32
	Projecting	Data	1.210	1.892	0.889	0.480	0.577	78.68
		Weibull	1.194	1.863	0.879	0.478	0.579	78.74
31	Original	Data	1.754	2.531	1.342	0.423	0.261	81.67
		Weibull	1.738	2.504	1.331	0.418	0.266	81.56
	Sampled	Data	1.782	2.525	1.377	0.401	0.245	81.91
		Weibull	1.767	2.499	1.366	0.396	0.249	81.79
	Projecting	Data	1.271	1.951	0.942	0.464	0.530	79.17
		Weibull	1.267	1.949	0.938	0.466	0.531	79.07
33	Original	Data	1.873	2.807	1.405	0.458	0.267	80.74
		Weibull	1.845	2.764	1.384	0.456	0.275	80.64
	Sampled	Data	1.937	2.786	1.484	0.414	0.221	81.42
		Weibull	1.920	2.764	1.471	0.413	0.227	81.34
	Projecting	Data	1.385	2.137	1.025	0.468	0.465	79.04
		Weibull	1.377	2.125	1.018	0.469	0.467	79.04
34	Original	Data	2.041	2.928	1.567	0.419	0.208	81.94
		Weibull	2.043	2.943	1.565	0.422	0.211	81.76
	Sampled	Data	2.071	2.907	1.608	0.392	0.182	82.21
		Weibull	2.074	2.915	1.609	0.392	0.186	82.06
	Projecting	Data	1.424	2.184	1.057	0.464	0.440	79.28
		Weibull	1.415	2.170	1.049	0.463	0.443	79.14
35	Original	Data	2.058	2.967	1.575	0.419	0.206	81.54
		Weibull	2.056	2.971	1.572	0.420	0.210	81.42
	Sampled	Data	1.999	2.881	1.530	0.415	0.214	81.35
		Weibull	1.997	2.889	1.526	0.418	0.218	81.22
	Projecting	Data	1.439	2.237	1.061	0.476	0.440	78.89
		Weibull	1.431	2.229	1.054	0.476	0.444	78.79
36	Original	Data	2.145	3.132	1.631	0.436	0.208	81.53
		Weibull	2.149	3.167	1.626	0.447	0.215	81.40
	Sampled	Data	2.151	3.074	1.654	0.408	0.185	81.63
		Weibull	2.122	3.028	1.634	0.406	0.188	81.62
	Projecting	Data	1.456	2.272	1.071	0.479	0.435	78.82
		Weibull	1.445	2.262	1.062	0.481	0.439	78.67
37	Original	Data	2.176	3.194	1.650	0.441	0.205	81.42
		Weibull	2.168	3.189	1.642	0.442	0.211	81.30
	Sampled	Data	2.255	3.203	1.740	0.403	0.165	81.87
		Weibull	2.253	3.201	1.738	0.402	0.168	81.75
	Projecting	Data	1.508	2.358	1.108	0.482	0.411	78.77
		Weibull	1.501	2.352	1.103	0.482	0.414	78.69
38	Original	Data	2.284	3.301	1.746	0.432	0.192	82.07
		Weibull	2.298	3.335	1.753	0.435	0.197	81.96
	Sampled	Data	2.324	3.293	1.796	0.404	0.164	82.11
		Weibull	2.371	3.388	1.824	0.412	0.166	81.89
	Projecting	Data	1.588	2.447	1.176	0.466	0.368	79.02
		Weibull	1.584	2.447	1.171	0.469	0.371	79.01

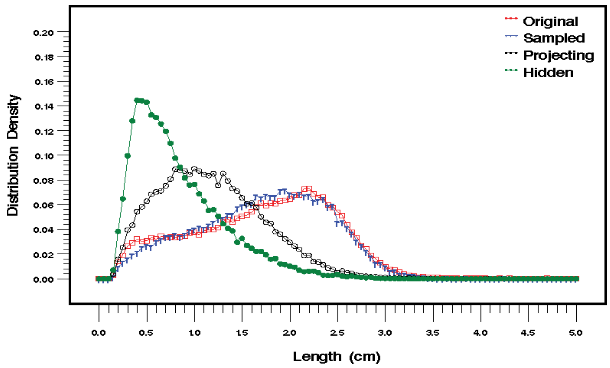

Figure 2. PDFs (by number) of the original sample, fibers picked by the HVI clamp, projecting portion of fibers, and the hidden portion of fibers.

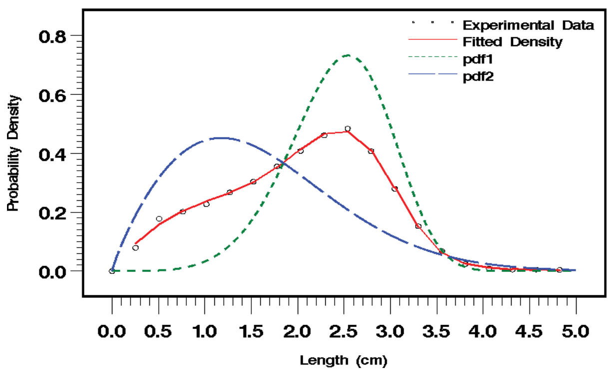

Figure 3. Probability density functions (by number) of ID 34 original fibers

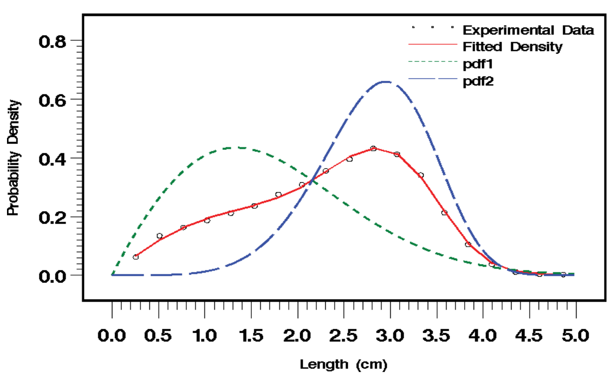

Figure 4. Probability density functions (by number) of ID 34 HVI sampled fibers

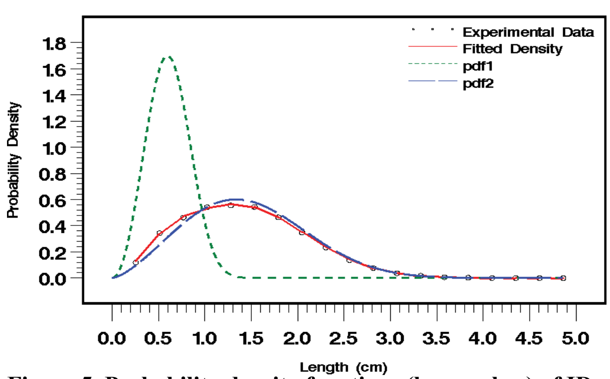

Figure 5. Probability density functions (by number) of ID 34 projecting fibers

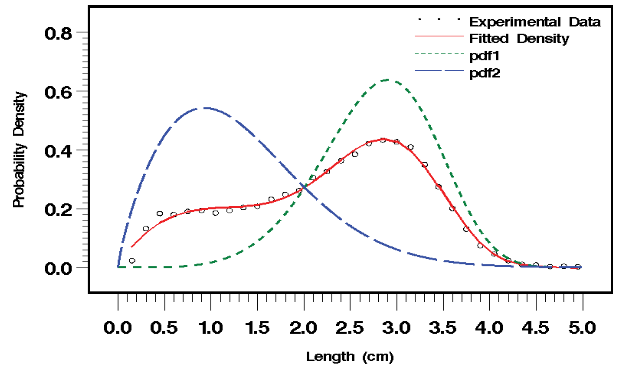

Figure 6. Probability density functions (by number) of ID 38 original fibers

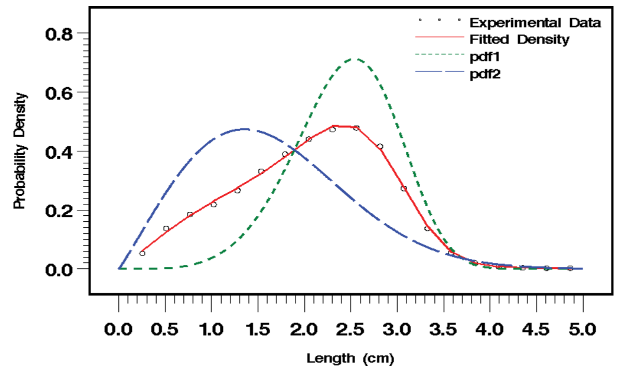

Figure 7. Probability density functions (by number) of ID 38 HVI sampled fibers

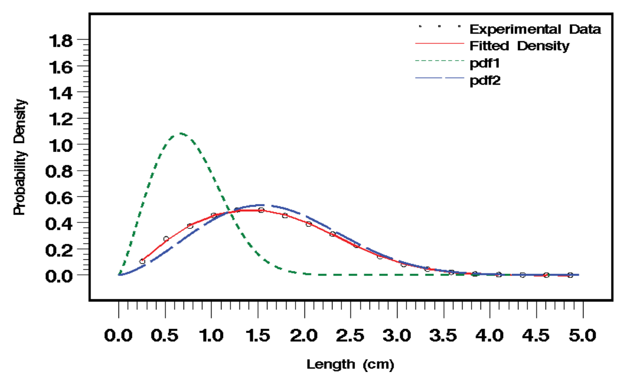

Figure 8. Probability density functions (by number) of ID 38 projecting fibers

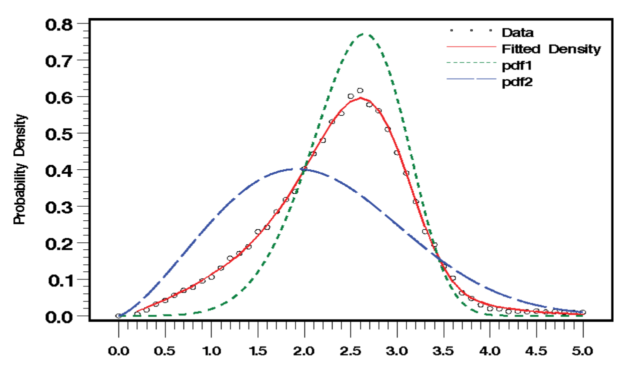

Figure 9. Probability density functions by weight of ID 34 original fibers

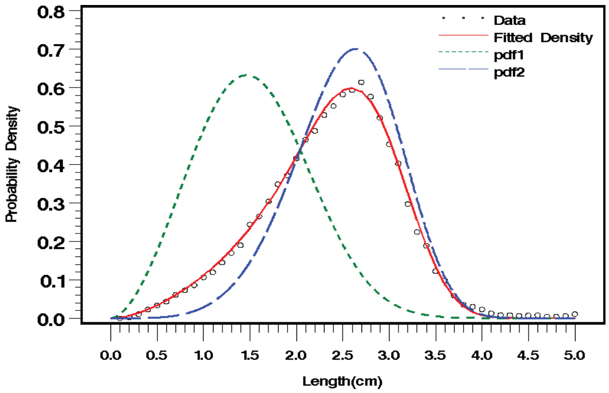

Figure 10. Probability density functions (by weight) of ID 34 HVI sampled fibers

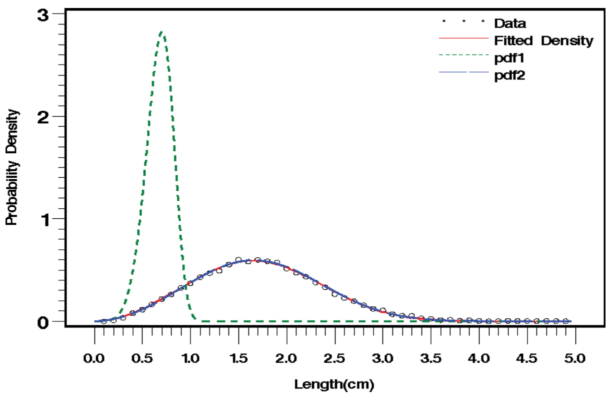

Figure 11. Probability density functions (by weight) of ID 34 projecting fibers

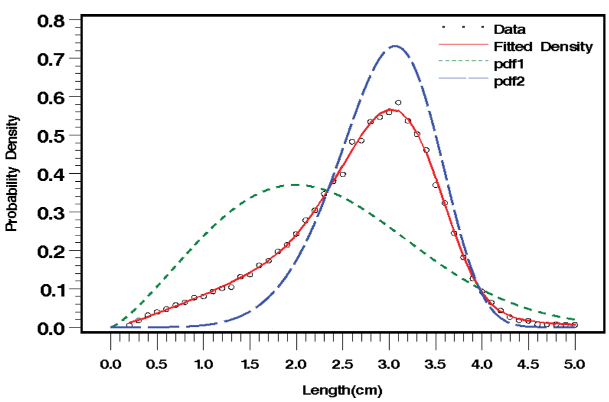

Figure 12. Probability density functions (by weight) of ID 38 original fibers

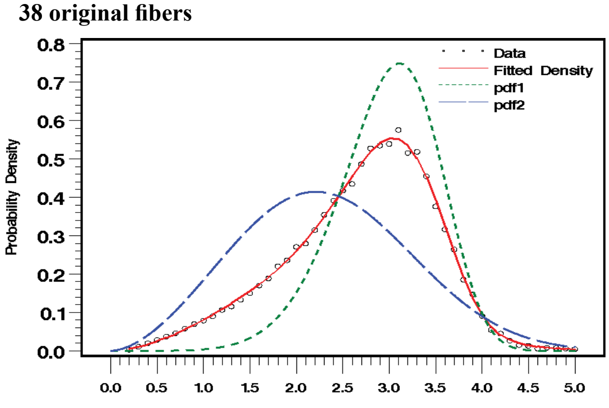

Figure 13. Probability density functions (by weight) of ID 38 HVI sampled fibers

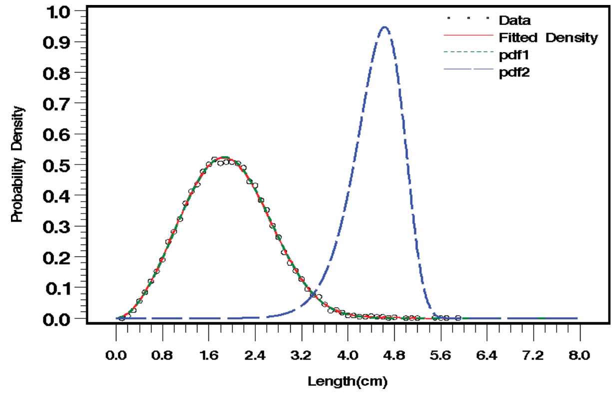

Figure 14. Probability density functions (by weight) of ID 38 projecting fibers

CONCLUSIONS

The theoretical distribution functions that can describe the underlying distributions of three types of fiber lengths that are related to HVI measurements were studied: the lengths of the original fiber population; the fibers picked by the HVI fibrosampler, and of the beard's projecting portion that is actually scanned by HVI.

Non-linear regressions were conducted based on length data measured using AFIS from eight cottons. A mixture of two Weibull distributions fits the data very well. Kolmogorov-Smirnov goodness-of-fit test confirms that a mixed Weibull distribution can be used as the underlying distribution of fiber length. The length parameters, such as ML and UHML, calculated from the mixture of Weibull distribution also matched those calculated from the actual test data extremely well.

Since the distribution of fiber length can be described as a mixture of two Weibull distributions which in turn is determined by five parameters, the relationship between the length distribution of projecting fibers and that of the original fibers can be investigated by exploring the relationship between the five parameters of the mixture Weibull distributions of them, which will be discussed in a subsequent paper.

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DISCLAIMER

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