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COMPARISON OF YARN STRENGTH PREDICTIONS WITH USTER STATISTICS

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Abstract

The prediction of yarn strength based on the three-stage procedure and assumption of fiber strength distribution of the Weibull type is compared with the empirical Solověv model. The predictive ability is compared with USTER Statistics. The sensitivity of both models to the relative variation of input data is estimated.

Introduction

Yarn mechanical properties are important for prediction of fabric mechanical behavior and estimation of yarn complex quality. The majority of models for yarn prediction is based on fiber characteristics only and is valid for a restricted range of fibers and yarns. In this contribution two techniques were selected, including the parameters of yarn formation as well. Computer experiments are used for evaluation of sensitivity of these models to the variation of input variables. The predictive ability of both models is compared with USTER Statistics.

Prediction of Yarn Strength Based on Weibull Distribution

This approach to yarn strength is used in the works (Pan 1992) for prediction of classical one-component and hybrid two-component yarns. The following assumptions are used:

1. The fiber strength has a two-parameter Weibull distribution.
2. The fiber helix angles in the yarn are randomly distributed from zero to the value at the yarn surface. Fiber migration is negligible.
3. Yarn twist level does not alter the yarn strength distribution.
4. When a fiber breaks, the load is carryied by survived fibers (distributed equally among the rest of fibers).
5. The changes of yarn geometry and dimensions during extension are neglected.

The computation has three stages:

1. Estimation of fiber Weibull parameters from experimental mean strength and standard deviation.
2. Estimation of parallel fibrous bundle of fibers strength.
3. Estimation of twisted bundle strength (yarn strength).

Let the fiber distribution be of the Weibull two-parameter type (Pan 1993)

$$F(\delta) = 1 - \exp\left(-l_y.\alpha_y.\delta_y^{\beta_y}\right) \tag{1}$$

The symbol denotes $1-F(\delta)$ probability of fiber surviving at strengths $x \le \delta$.

In eqn. (1), l_y [mm] is fiber length, α_y is a scale parameter, and β_y is a shape parameter. The mean fiber strength $\overline{\delta_y}$ and corresponding standard deviation $s_{\overline{\delta y}}$ are computed from equations:

$$\overline{\delta_y} = \left(l_y.\alpha_y\right)^{-\frac{1}{\beta_y}} \Gamma\left(1+\frac{1}{\beta_y}\right) \text{ [N.tex}^{-1}\text{]} \tag{2}$$

$$s_{\overline{\delta y}} = \overline{\delta_y}\left(\frac{\Gamma\left(1+\frac{2}{\beta_y}\right)}{\Gamma^2\left(1+\frac{1}{\beta_y}\right)}-1\right)^{\frac{1}{2}} \text{ [N.tex}^{-1}\text{]} \tag{3}$$

where $\Gamma()$ is the gamma function. For known $\overline{\delta_y}$ a $s_{\overline{\delta y}}$ computed from experimental data is parameter β_y evaluated by iterative solution of the equation:

$$\overline{\delta_y}\left(\frac{\Gamma\left(1+\frac{2}{\beta_y}\right)}{\Gamma^2\left(1+\frac{1}{\beta_y}\right)}-1\right)^{\frac{1}{2}}-s_{\overline{\delta y}}=0 \tag{4}$$

in the interval $\beta \in \langle 1;9 \rangle$. The estimated β_y is then substituted to equation (2) and from this equation is evaluated $(l_y.\alpha_y)$.

For the computation of fibrous bundles distribution it is possible to use Daniel's famous result, that for large bundles (number of fibers in cross section N_y is more than 100) the bundle strength approaches a normal distribution (Pan 1993):

$$H(\delta_b)=\frac{1}{\sqrt{2\pi}.s_{\overline{\delta b}}}\exp\left[-\frac{\left(\delta_b-\overline{\delta_b}\right)^2}{2s_{\overline{\delta b}}^{\ 2}}\right] \tag{5}$$

Mean strength, $\overline{\delta_b}$, of the fibrous bundle is equal to :

$$\overline{\delta_b}=\left(l_y\alpha_y\beta_y\right)^{-\frac{1}{\beta_y}}.\exp\left[1-\frac{1}{\beta_y}\right]\text{[N.tex}^{-1}\text{]} \tag{6}$$

The corresponding standard deviation $s_{\overline{\delta b}}$ is

$$s_{\overline{\delta b}}=\left(l_y\alpha_y\beta_y\right)^{-\frac{2}{\beta_y}}.\exp\left(-\frac{1}{\beta_y}\right).\left(1-\exp\frac{1}{\beta_y}\right).N_y^{-1}\ \text{[N.tex}^{-1}\text{]} \tag{7}$$

Once a twist is inserted to the fibrous bundle, the fibers are entangled with each other. During tensile deformation the lateral pressure is induced. For the evaluation of twist influence on changes of fibrous bundle strength the nominal yarn surface helix angle q (Hearle 1969) is required.

$$q=arctg\left(10^{-3}T_y\sqrt{\frac{40\pi}{\rho_f V_f}}\right)\text{[rad]} \tag{8}$$

where ρ_f [kg.m^{-3}] is fiber density, V_f is fiber volume fraction and T_y is yarn twist factor defined as

$$T_y=\sqrt{T}.Z\ \text{[tex.cm}^{-1}\text{]} \tag{9}$$

Here T [tex] is yarn fineness and Z [cm^{-1}] is number of twists. It was found that fiber volume factor could be expressed as

$$V_f=0.7-(1-0.78*\exp(-0.195*T_y)) \tag{10}$$

Angle β is computed from twist intensity $tg\beta$ according to the relation:

$$\beta=arctg\left(\pi DZ\right)=arctg\left(\pi Z\sqrt{\frac{4T}{\pi\mu\rho}}\right)=arctg\left(Z\sqrt{T}\sqrt{\frac{4\pi^2}{\pi\mu\rho}}\right)=arctg\left(T_y\sqrt{\frac{4\pi}{\mu\rho}}\right)\text{[rad]} \tag{11}$$

Poisson's ratio of twisted bundle ν_{lt} is dependent on the yarn surface helix only

$$\nu_{lt} = \frac{\sin^2 q}{2\left(1-\cos^3 q\right)\left(\frac{1}{2q}-\frac{1}{4}\sin q\right)} \text{ [1]} \tag{12}$$

Orientation efficiency factor η_q has the form

$$\eta_q = \frac{2q\left(1-\nu_{lt}\right)+\left(1+\nu_{lt}\right)\sin 2q}{4q} \text{ [1]} \tag{13}$$

In the case of parallel fiber bundle is q approaching to zero and fiber strain at break. The final equation for yarn strength $\overline{\delta_s}$ has simple form

$$\overline{\delta_s} = V_f.\eta_f.\overline{\delta_b} \text{ [N.tex}^{-1}\text{]} \tag{14}$$

where $\overline{\delta_b}$ [N.tex^{-1}] is mean strength of parallel fibrous bundle. Standard deviation of yarn strength, $s_{\overline{\delta_s}}$, is given by:

$$s_{\overline{\delta_s}} = V_f.\eta_f.s_{\overline{\delta_b}} \text{ [1]} \tag{15}$$

where $s_{\overline{\delta b}}$ [N.tex^{-1}] is standard deviation of strength in parallel bundle. Inclusion of lateral interaction is described in (Pan 1992). The distribution of yarn strength is assumed to be normal as well and fully defined by parameters $\overline{\delta_s}$ and $s_{\overline{\delta_s}}$.

Prediction of Yarn Strength Based on Solověv Model

This empirical approach is using a set of coefficients modifying the relation between fiber strength and yarn strength. Yarn strength F [N.tex^{-1}] is defined as fiber strength F_v [N.tex^{-1}] reduced by factors dependent on the fiber characteristics and twist level.

$$F = F_v.f_n.f_l.f_\alpha.\eta \text{ [N.tex}^{-1}\text{]} \tag{16}$$

Here factor f_n is characterizing of number of fibers influence, f_l is characterizing of fiber length and length variation and f_α is characterizing of twist level influence. Solov v proposed for f_n empirical form

$$f_n = 1 - C.H - \frac{K}{\sqrt{n}} \text{ [1]} \tag{17}$$

where C is constant (for cotton is 0,375), H is correction for technology used (combed yarns 3,5-4, carded 4,5-5) and K *is* constant (for cotton is 2,65). Number of fibers in cross section n is computed from well-known ratio

$$n = \frac{T}{t} \text{ [1]} \tag{18}$$

where T is yarn fineness and t is fiber fineness (both in [tex]). For the factor f_l the following relation could be used:

$$f_l = 1 - \frac{a}{l} \text{ [1]} \tag{19}$$

where a is empirical constant (for cotton 5) and l [mm] is fiber length.. For computation of f_α it is necessary to derive Koechlin's twist factor α and critical Koechlin's twist factor α_k. Koechlin's critical twist factor is computed from the relation:

$$\alpha_k = \left(55.T^{0,0908}\right).\frac{\sqrt{10}}{\sqrt[6]{T}} \text{ [m}^{-1}\text{ktex}^{1/2}] \tag{20}$$

Koechlin's twist factor is derived as:

$$\alpha = Z.\sqrt{\frac{T}{1000}} \text{ [m}^{-1}\text{ktex}^{1/2}] \tag{21}$$

The difference between these factors is denoted as δ_α , and is given by:

$$\delta_\alpha = \alpha - \alpha_k \text{ [m}^{-1}\text{ktex}^{1/2}] \tag{22}$$

The regression type equation is proposed for computation of f_α (Zelinková, Brzezina 2000).

$$f_\alpha = 1 + \delta_\alpha^{\,2}\left[6,67.10^{-7}.\delta_\alpha - \frac{0,02027}{179,4+\left(\delta_\alpha+10\right)^2} - 8.10^{-5}\right] [1] \tag{23}$$

Quantity η lies in the interval 0,95-1,1. (for computations the value 1 is used) (Neckář 1969).

Experiment

Sensitivity of Models on Input Data Variation

The sensitivity of predicted yarn strength on the input characteristics changes was evaluated by computational experiments. Sensitivity is defined as the percent deviation of model yarn strength F from experimental yarn strength F_E:

$$Sensitivity = \frac{F - F_E}{F_E} * 100$$

Experimentally evaluated characteristics for combed ring cotton yarn were used as mean values for sensitivity evaluation. These characteristics are summarized in table 1. In each computational run the values of one parameter only were changed in prescribed range defined in table 2. Results of computational experiments are summarized on the fig 1- 5.

Comparison with USTER Statistics

For comparison and evaluation of predictive ability, the six real combed and carded cotton type yarns were prepared. Characteristic of these yarns are given in Table 3. For each yarn yarns strengths were computed for both models and included with experimentally measured yarn strengths to the graph having 5% and 95 % lines of corresponding USTER Statistics. These USTER Statistics curves were obtained by piecewise linear approximation from graphical forms. Results are shown in Figures 6 and 7.

For selected yarn types (carded and combed), predicted values of yarn strength for intervals of fiber fineness were computed. For carded yarn, the interval 16 - 18 tex was selected; and for carded yarn the interval 6 - 33 tex. The input parameters for both yarns are in Table 4. Results are shown on the Figures 8 and 9.

Conclusion

From all graphs is clear that both models are able to predict yarn strength with relatively great precision. The Uster Statistics 95% level is near both models but the Solovev model has higher values of prediction. On the other hand, the Weibull type model is often very close to the experimental values. From sensitivity analysis it is clear that both models are similarly sensitive on the changes of fiber strength and yarn fineness. In other parameters, there are differences in sensitivity caused mainly by the simplified assumptions in the Weibull model.

References

Hearle, J.W., Grosberg, P., and Backer, S. 1969. Structural mechanics of yarns and fabrics. Vol 1, Interscience, NY

Neckář, B. 1990. Příze. SNTL, Praha, .

Pan, N. 1992. Text. Res. J., **62**, 749.

Pan, N. 1993. J. Mater. Sci. **28**, 6107 ().

Zelinková, K. and Brzezina, M. 2000. Ověřování metod pro predikci pevností přízí. TU v Liberci, Liberec, .

Table 1. Input data.

Fiber length [mm]	Fiber fineness [tex]	Fiber density [kg/m3]	Fiber strength [cN/tex]	Fiber break strain [%]	Yarn fineness [tex]	Yarn twist [1/m]
24,86	0,12	1520	35,69	4,83	7	1220

Table 2. Intervals for sensitivity evaluation.

Range	Fiber length [mm]	Fiber fineness [tex]	Fiber strength [cN/tex]	Fiber break strain [%]	Yarn fineness [tex]	Yarn twist [1/m]
min	0,1	20	13,605	3	5	600
max	0,4	34,5	37,415	10,2	27,5	1200

Table 3. Experimental data for selected yarns.

Yarn type	Fiber length [mm]	Fiber fineness [tex]	Fiber density [kg/m^3]	Fiber strength [cN/tex]	Fiber break strain [%]	Yarn fineness [tex]	Yarn twist [1/m]
prst7,4 MIIces	24,86	0,12	1520	35,69	4,83	7,1	1220
Prst10MIIces	24,86	0,148	1520	35,69	4,83	9,84	1291
prst16.5A1ces	24,24	0,155	1520	29,56	6	15,92	972
Prst20AImyk	23,16	0,155	1520	25,90	5,74	19,27	748
prst295AImyk	23,16	0,165	1520	25,90	5,74	29,52	630
prst 38AImyk	23,16	0,165	1520	25,90	5,74	37,3	533

Table 4. Input parameters for prediction of ring yarn strength

Yarn type	Fiber Length [mm]	Fiber fineness [tex]	Fiber strength [cN/tex]	Fiber break strain [%]	Yarn twist [1/m]
Prst20AImyk	23,16	0,155	25,90	5,74	748
prst7,4 MIIces	24,86	0,12	35,69	4,83	1220

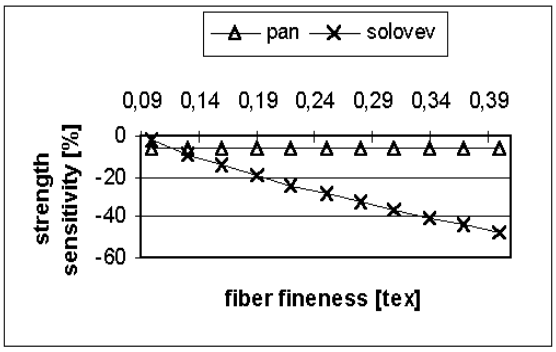


Figure 1. Sensitivity on fiber fineness.

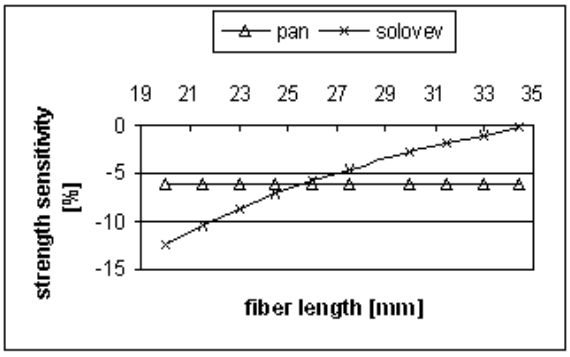


Figure 2. Sensitivity on fiber length.

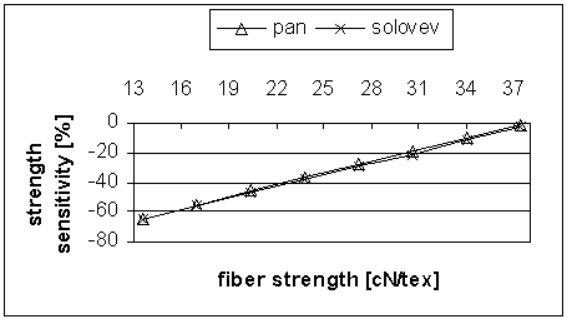


Figure 3. Sensitivity on fiber strength.

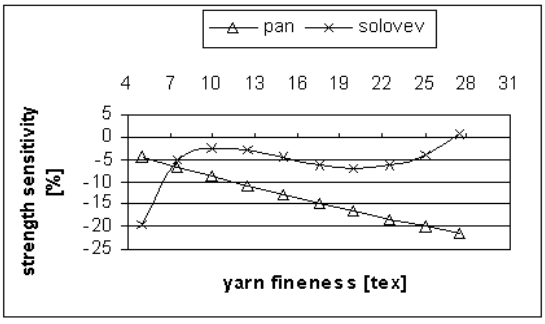


Figure 4. Sensitivity on yarn fineness.

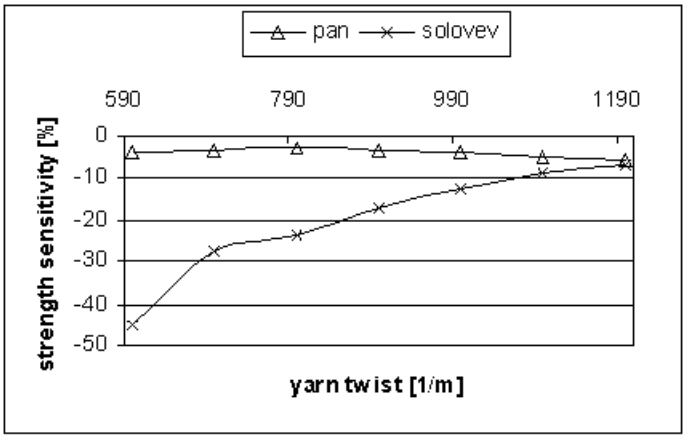
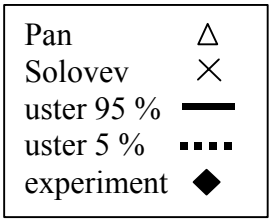


Figure 5. Sensitivity on yarn twist.



Legends to figures:

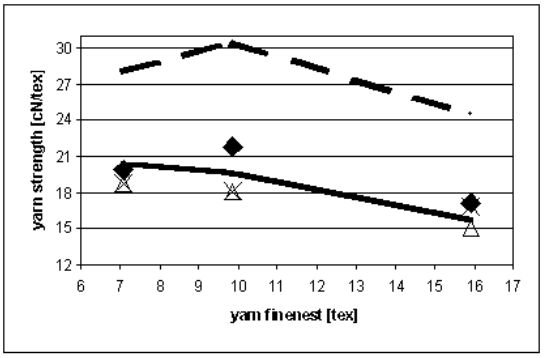


Figure 6. Combed ring yarns.

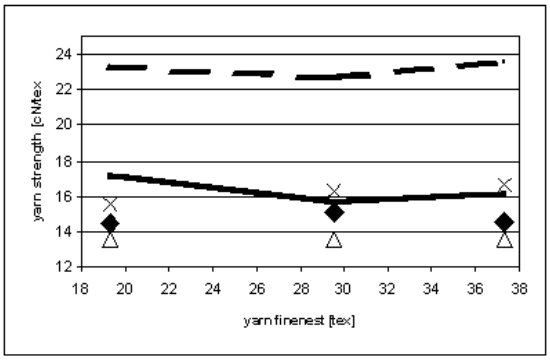


Figure 7. Carded ring yarns.

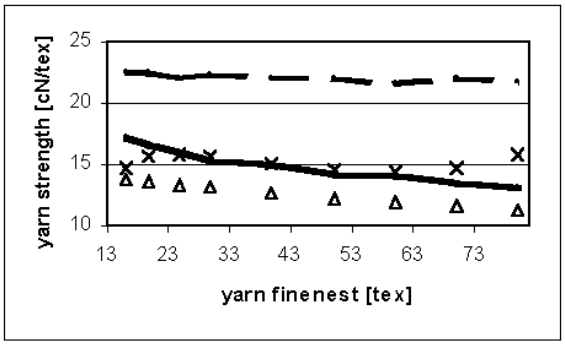


Figure 8. Carded ring yarn.

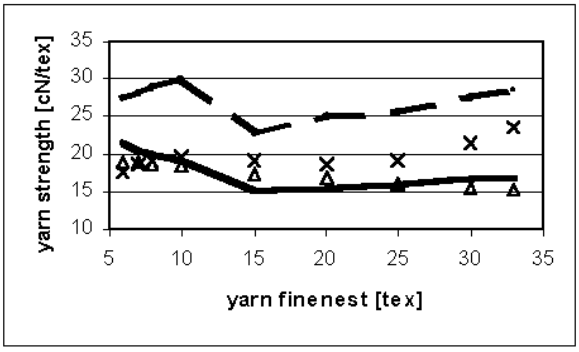


Figure 9. Combed ring yarn.