THEORETICAL STUDY OF THE PARTICLE COLLECTION MECHANISM IN THE OUTER VORTEX OF A CYCLONE

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Abstract

This paper presents a new theoretical analysis of the particle collection mechanism in the outer vortex of a cyclone. The radial component of the particle trajectory model is developed based on a force balance differential equation. The particle (with diameter of d_{50}) collection probability is 50% when it is under the force balance condition. The d_{50} distribution in the space of the outer vortex is mathematically determined by solving the force balance differential equation. The particle collection probability distribution in the outer vortex is obtained based upon the inlet particle size distribution and the d_{50} distribution. The critical separating diameter d_{50} distribution and particle collection probability distribution in the outer vortex are the bases to simulate the collection efficiency and to predict the emission concentrations.

Introduction

Study of the particle collection mechanism in the outer vortex is a way to understand the relationship between the cyclone performance characteristics and the design parameters and then to develop a mathematical model to estimate the cyclone performance. This study is based upon the knowledge of the flow pattern in the cyclone outer vortex. Many investigations have been made to determine the flow pattern (velocity profile) in a cyclone irrotational fluid field. Shepherd and Lapple (1939) reported that the primary flow pattern consisted of an outer spiral moving downward from the cyclone inlet and an inner spiral of smaller radius moving upward into exit pipe. These flows are known as outer vortex and inner vortex. The transfer of fluid from the outer vortex to the inner vortex apparently begins below the bottom of the exit tube and continues down into the cone to a point near the dust outlet at the bottom of the cyclone. Shepherd and Lapple concluded from streamer and pitot tube determinations that the radius marking the outer limit of the inner vortex and the inner limit of the outer vortex was roughly equal to the exit duct radius. Ter Linden (1949) measured the details of the flow field in a 14 in cyclone. He reported that the interface of inner vortex and outer vortex occurred as a radius somewhat less than that of the exit duct in the cylindrical section of the cyclone and approached the centerline in the conical section.

The velocity profile in a cyclone can be characterized by three components (tangential, axial and radial). The tangential velocity is the dominant component. It also determines the centrifugal force applied to the air stream. Research results of Shepherd and Lapple (1939), Ter Linden (1949) and Ma (1983) indicated that air stream tangential velocity in the annular section (at the same cross-sectional area) of the cyclone could be determined by equation (1).

$$V_t * r^n = C_1 \tag{1}$$

where:

- V_t = tangential velocity,
- r = air stream radius,
- n = 0.5 (in outer spiral) (Shepherd and Lapple, 1939), and
- C_1 = a numerical constant.

For simplicity, it is assumed that V_t equals the average air stream inlet velocity V_{in} when r equals the radius of the cyclone wall (R), that is:

$$V_t * r^{0.5} = V_{in} * R^{0.5}$$

$$V_t = \left(\frac{R}{r}\right)^{0.5} * V_{in} \tag{2}$$

where:

V_t = tangential velocity,
V_{in} = inlet velocity,
R = the radius of the cyclone wall, and
R = air stream radius.

Theoretical Analyses of the Particle Collection Mechanism in the Outer Vortex

Particle Motion and Its Differential Equation in the Outer Vortex

In the study of the particle trajectory in the cyclone outer vortex, the following assumptions were made:

1. The relative velocity between air the particle does not change the fluid pattern (air stream velocity profile) in the outer vortex.
2. The particle is spherical.
3. The particle motion is not influenced by the neighboring particles.
4. The drag force on the particle is given by Stokes' Law
5. The air stream radial velocity is zero and tangential velocity is given by equation (2).
6. The particle tangential velocity is the same as the air stream tangential velocity (Leith, 1972).
7. The particle moves from the interface of inner vortex and outer vortex towards the cyclone wall, once the particle hits the wall, it will be collected.
8. A force balance on a particle yields 50% collection probability on this particle.

When a particle with diameter d and density ρ enters the cyclone outer vortex, its initial location (time t=0) is at the point A (r_o, θ_o, Z_o), which is on the interface of the inner vortex and outer vortex. Centrifugal force provides the particle with a radial acceleration to make the particle move from the interface to the cyclone wall. At the time t, the particle will move to the point B (r, θ, Z). After time t+Δt, the particle will move to the point C (r+Δr, θ+Δθ, z+Δz) on the cyclone wall (Figure 1). If the analysis is conduced in the cylindrical coordinate system, i.e. in **r θ** coordinates, the particle velocity vector is as follows:

$$\vec{V} = r\frac{d\theta}{dt}\vec{T} + \frac{dr}{dt}\vec{R} \tag{3}$$

where:

$\vec{V}$ = particle velocity vector,
$r\frac{d\theta}{dt}$ = particle tangential velocity (V_t),
$\frac{dr}{dt}$ = particle radial velocity (V_r),
$\vec{T}$ = tangential unit vector, and
$\vec{R}$ = radial unit vector.

The particle acceleration can be determined by equation (4):

$$\bar{a} = \frac{d\bar{V}}{dt} = \frac{\partial \bar{V}}{\partial \theta}\frac{\partial \theta}{\partial t} + \frac{\partial \bar{V}}{\partial r}\frac{\partial r}{\partial t}$$
$$\bar{a} = \left(r\frac{d^2\theta}{dt^2} + \frac{dr}{dt}\frac{d\theta}{dt} \right)\bar{T} + \left(\frac{d^2 r}{dt^2} - r(\frac{d\theta}{dt})^2 \right)\bar{R} \tag{4}$$

where:

$\left(r\frac{d^2\theta}{dt^2} + \frac{dr}{dt}\frac{d\theta}{dt} \right)$ = particle tangential acceleration (a_t), and

$\left(\frac{d^2 r}{dt^2} - r(\frac{d\theta}{dt})^2 \right)$ = particle radial acceleration (a_r)

Since the particle tangential velocity is equal to the air stream tangential velocity at a constant value for the given radius r, there is no tangential acceleration ($a_t = 0$). As a result, the inertial force (centrifugal force) F_c can be determined by the following equation:

$$F_c = m\bar{a} = m\bar{a}_r = \frac{\pi * d^3}{6} * \rho * \left(\frac{d^2 r}{dt^2} - r(\frac{d\theta}{dt})^2 \right) \tag{5}$$

where:

F_c = centrifugal force acting on the particle in radial direction,
M = particle mass,
ρ = particle density, and
d = particle diameter.

Along the radial direction, there is another force working on the particle, i.e. the drag force. The drag force (F_D) is determined by Stokes' Law as:

$$F_D = 3\pi\mu d * (V_r - V_{gr}) = 3\pi\mu d * (\frac{dr}{dt} - V_{gr}) \tag{6}$$

where:

F_D = drag force acting on the particle,
μ = air viscosity,
V_r = particle radial velocity, and
V_{gr} = air stream radial velocity.

In the cyclone outer vortex fluid field, there are only two forces (F_c & F_D) acting on the particle in the radial direction. When $F_c > F_D$, the particle moves towards the cyclone wall to be collected. Whereas, when $F_c < F_D$, the particle will move to the inner vortex and then to penetrate the cyclone. The force balance ($F_c = F_D$) gives the particle a 50% chance to penetrate and a

50% chance to be collected. The force balance differential equation can be setup by letting equation (5) equal to equation (6), i.e. $F_c = -F_D$, it yields the equation (7).

$$\left(\frac{d^2 r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 \right) = -\frac{18\mu}{\rho * d^2} * \left(\frac{dr}{dt} - V_{gr} \right) \tag{7}$$

where:

$\frac{dr}{dt}$ = particle radial velocity (V_r),

$r\frac{d\theta}{dt}$ = particle tangential velocity (V_t can be determined by the equation (2)),

V_{gr} = air stream radial velocity = 0,

μ = air viscosity,

ρ = particle density, and

d = particle diameter.

Let $K = \frac{18\mu}{\rho * d^2}$, then the equation (7) can be rewritten in the following form:

$$\frac{d^2 r}{dt^2} + K * \frac{dr}{dt} - \frac{V_t^2}{r} = 0 \quad (r_o < r < R) \tag{8}$$

where:

r = particle radial location,

t = particle traveling time,

V_t = particle tangential velocity (determined by equation (2)),

r_o = radius of the interface of the inner vortex and outer vortex, and

R = radius of the cyclone wall.

Equation (8) is the particle force balance differential equation in the outer vortex.

Particle Trajectory in the Outer Vortex

The solution of equation (8) gives the particle radial trajectory (critical path in the radial direction) in the r θ coordinates. To solve the equation, the following initial conditions are used:

r = r_o at t = 0 and r_o = radius of the interface of the inner vortex and outer vortex = radius of the exit tube

$\frac{dr}{dt}$ = V_r, and V_r = 0 at t = 0

On the other hand, the particle trajectory in rz coordinate is more concerned. So the differential equation (8) should also be solved in rz coordinate. It is assumed that the particle motion in the rz coordinates follows a linear path. As a result, the acceleration term d^2r/dt^2 can be neglected in the differential equation (8). And then, the V_t term in the equation (2) is put into equation (8), the following differential equation is obtained:

$$K * \frac{dr}{dt} = \frac{R * V_{in}^2}{r^2} \qquad (r_o < r < R) \tag{9}$$

where:

$t = \frac{Z}{V_z}$ = particle traveling time in the Z distance along the axial direction,

$dt = \frac{dZ}{V_z}$ = particle traveling time in the dZ distance along the axial direction, and

$V_z = \frac{2}{3\pi} V_{in}$ = particle and air stream axial velocity component (for 1D3D and 2D2D design) (Wang, 2001)

The solution of equation (9) gives the particle motion trajectory in the rz plane as the following:

$$r(z) = \sqrt[3]{r_o^3 + \frac{\rho * \pi * R * V_{in} * d^2}{4\mu} * Z} \tag{10}$$

where:

- r = particle radial travel distance,
- r_o = interface radius = radius of the exit tube = $D_e/2$ = D/4 (for 1D3D and 2D2D design),
- D_e = cyclone exit tube diameter,
- D = cyclone diameter,
- R = radius of cyclone wall = D/2,
- d = particle diameter,
- ρ = particle density,
- μ = air viscosity
- Z = particle travel distance along axial direction, and
- V_{in} = cyclone inlet velocity.

Equation (10) is the particle radial trajectory function in the outer vortex.

D_{50} Distribution in the Outer Vortex

As mentioned above, the force balance on the particle gives the particle a 50% chance to be collected and a 50% chance to penetrate. In other words, the collection efficiency on this particle will be 50% when the particle is under the force balance condition. It is notated that the particle diameter is d_{50} when the particle is at the force balance situation. In fact, d_{50} is the critical separating diameter. If a particle larger than d_{50}, it will move towards the wall, whereas, if a particle is smaller than the d_{50}, it will move towards inner vortex. The particle diameter (d) is the critical separating diameter (d_{50}) in equation (10). Studying this equation, it is observed that in the cyclone outer vortex space, there is a d_{50} distribution. This distribution is the function of the location (r, z), particle density, cyclone design and inlet velocity. The d_{50} distribution function can be obtained by rewriting equation (10) as the following:

$$d_{50} = \sqrt{\frac{4\mu * (r^3 - r_o^3)}{\rho * \pi * R * V_{in} * Z}} \tag{11}$$

where:

- d_{50} = particle diameter with a force balance,
- r = particle radial location at point P(r,z),
- r_o = interface radius = radius of the exit tube = $D_e/2$ = D/4 (for 1D3D and 2D2D design),
- D_e = cyclone exit tube diameter,
- D = cyclone diameter,
- R = radius of cyclone wall = D/2,
- ρ = particle density,
- μ = air viscosity
- Z = particle axial location at point P(r,z), and
- V_{in} = cyclone inlet velocity.

Equation (11) is the d_{50} distribution in the outer vortex space.

The Particle Collection Probability Distribution in the Outer Vortex

Based on the analyses above, d_{50} distribution gives the critical separation diameter (d_{50}) at the any point P(r, z) in the outer vortex. At the point P(r, z), if the particle diameter $d > d_{50}$, the particle will move to the wall and be collected, whereas if the particle diameter $d < d_{50}$, the particle will move to the inner vortex and penetrate. For a given inlet particle size distribution, the ratio of all the particles larger than d_{50} to the total inlet particles is the particle collection probability at the point P(r, z). If it is assumed that inlet particle size distribution is a log-normal distribution with mass median diameter MMD and geometric standard deviation GSD, then equation (12) can be used to determine the particle collection probability at any point P(r, z) in the outer vortex.

$$P(d) = \int_{d_{50}}^{\infty} \frac{1}{\sqrt{2\pi}d\ \ln(GSD)} \exp[-\frac{(\ln(d) - \ln(MMD))^2}{2(\ln(GSD))^2}]dd \qquad (12)$$

where:

P (d) = particle collection probability at the any point P(r, z) in the outer vortex,
d_{50} = the critical separation diameter with a 50% collection probability at the point P(r, z),
d = particle diameter,
MMD = mass median diameter of the inlet particle size distribution, and
GSD = geometric standard deviation of the inlet particle size distribution.

Equation (12) is the particle collection probability distribution in the outer vortex.

Sample Calculation and Discussion

Particle Trajectory Calculation

Equation (10) is the theoretical model to determine the particle trajectory in a cyclone outer vortex. The fly ash particles with density of 2.7 g/cm^3 and 36-inch 1D3D cyclone design are used in this sample calculation. For the 1D3D cyclone design, $r_o = D_e/2$, where D_e is the exit tube diameter. Putting all the given parameters into equation (10), it can be rewritten as the following:

$$r(z) = \sqrt[3]{0.01195 + 8.925456 * 10^8 * d^2 * Z} \qquad (13)$$

where:

r(z) = radial component of a particle trajectory (m),
z = axial component of a particle trajectory (m), and
d = particle diameter (m).

Figure 2. shows trajectories of the fly ash particles with diameters of 2.5µm, 10µm and 20µm in the outer vortex of a 36-in 1D3D cyclone. For the 36-in cyclone, the radius of the cyclone wall is 0.457m. From Figure 2., it is observed that the particles with diameter of 10µm and 20µm are much faster to reach the wall than the particle with diameter of 2.5µm. it means that the large particles could be collected at the very early time in the cyclone barrel part, whereas the smaller particle will be collected at the later time in the cyclone cone part.

D_{50} Distribution Calculations

The d_{50} distribution calculation is based on equation (11). Again, putting all the given parameters into equation (10), it can be rewritten as the following:

$$d_{50} = \sqrt{\frac{r^3 - 0.01195}{8.9254565 * 10^8 * Z}} \qquad (14)$$

where:

d_{50} = the critical separating diameter at the point P(r,z) in the outer vortex,
r = radial location of the particle at the point P, and
z = axial location of the particle at the point P.

Figure 3. gives d_{50} distribution curves in the cyclone outer vortex. Because of the tangential velocity (V_t) distribution (equation (1)) in the outer vortex the V_t reaches its maximum value on the interface, the centrifugal force decreases with the decrease of $V_{t.}$ As a result, the d_{50} increases with the increase of the radius of the radial location (r). On the other hand, d_{50} decreases with the increase of the axial location (z).

Future Research

The particle collection probability distribution is in fact the particle collection rate distribution in the outer vortex. It is also the collected concentration distribution in the outer vortex. The Monte Carlo simulation theory can be used to simulate the cyclone fractional efficiency curves based upon the particle collection probability distribution in the outer vortex and the inlet particle size distribution. The experimental test should be conducted to test the concentration distribution in the outer vortex and then the experimental collection probability distribution can be used to test the new theoretical equations in this paper.

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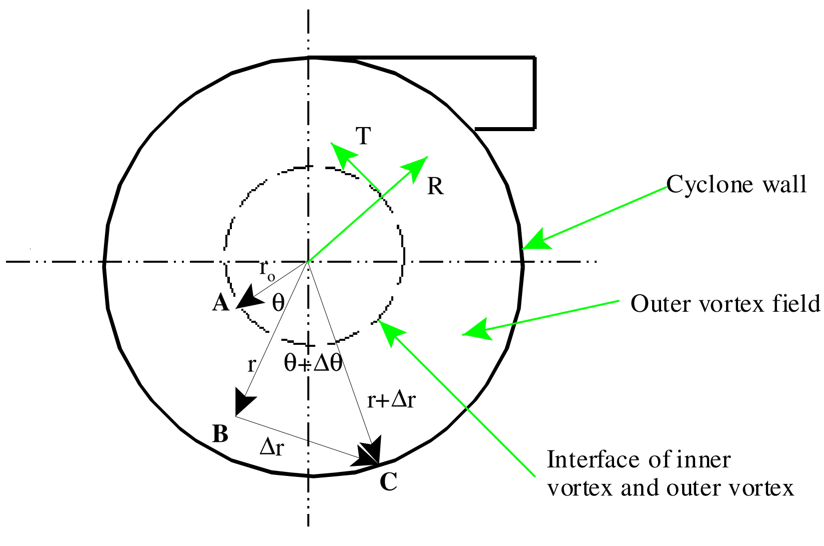

A (r_o, θ_o, z_o) at time = 0 **B** (r, θ, z) at time = t **C** (r+Δr, θ+Δθ, z+Δz) at time = t+Δt

Figure 1. Particle motion in the cyclone outer vortex.

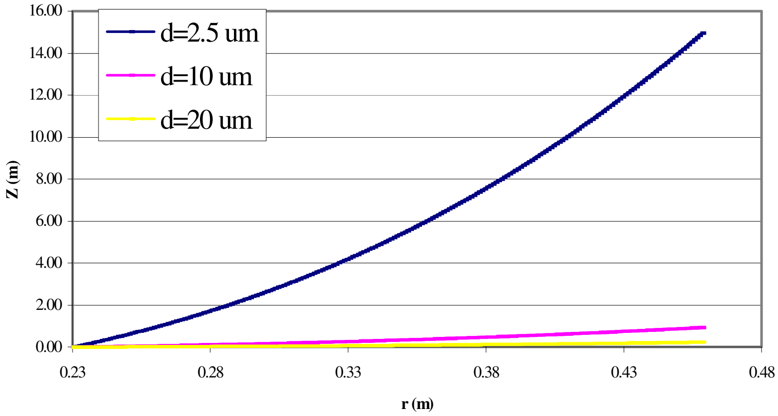

Figure 2. Fly ash particle trajectory in the outer vortex of a 36-in 1D3D cyclone.

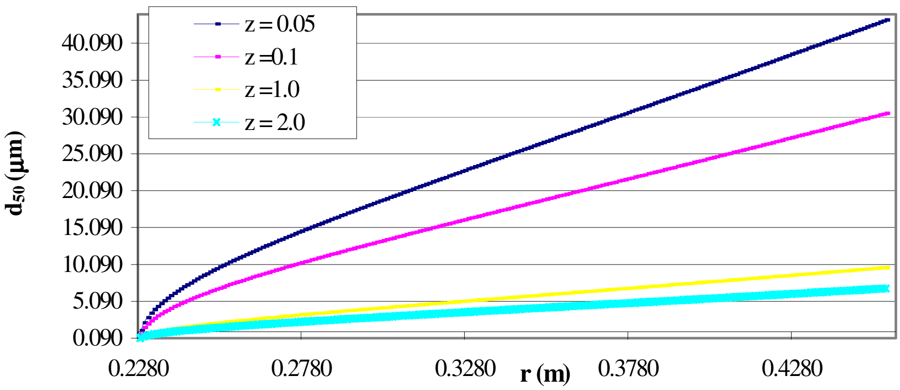

Figure 3. d_{50} distribution in the outer vortex of a 36-in 1D3D cyclone with fly ash dust.