# THE ECONOMICS OF SOIL FERTILITY UNDER PRECISION AGRICULTURE: THE CASE OF PHOSPHORUS

**Man Yu, Research Assistant**
**Department of Agricultural and Applied Economics**
**Texas Tech University**
**Eduardo Segarra, Professor**
**Department of Agricultural and Applied Economics**
**Texas Tech University**
**Arthur B. Onken, Professor**
**TAES - Lubbock**
**Texas A&M University**

## Abstract

A dynamic optimization model is used to derive and evaluate phosphorus fertilizer optimal decision rules for irrigated cotton production in the Southern High Plains of Texas. Results indicate that the optimal phosphorus application decision rules depend on initial phosphorus availability.

## Introduction

Increased use of fertilizer has been an important development in agriculture over the past several decades. Currently, production agriculture is facing significant challenges such as escalating costs of production and increased public concern about the impacts of agricultural production on the environment. As world trade liberalization continues, agricultural producers will compete to produce high quality products at low prices for the world market, while attempting to use production practices that are benign to the environment. Traditionally, optimal fertilizer input use in agriculture has assumed spatial field homogeneity with respect to fertility characteristics. That is, optimal fertilizer input decision rules do not account for fertility differences within fields. Precision farming, precision agriculture or site-specific management recognizes fertility variability within fields and seeks to optimize variable input use under these conditions. In this study, soil fertility in irrigated cotton production stemming from optimal phosphorus fertilizer application is addressed.

The primary objective of this study is to evaluate the economic implications of precision farming practices with respect to phosphorus application in irrigated cotton production in the Southern High Plains of Texas (SHPT). In particular, a dynamic optimization model of phosphorus utilization that introduces a dynamic phosphorus carry-over function is presented. The SHPT is a semiarid region located in the northwestern part of the state, which encompasses about 22 million acres (35,000 square miles) in 42 counties. Cotton is the most important crop produced in the area in terms of both acreage and crop value. Annual cotton plantings vary between 2.6 and 3.3 million acres in a 25-county region within SHPT, with approximately 50 percent of these acres under irrigation.

Reprinted from the *Proceedings of the Beltwide Cotton Conference*
Volume 1:289-292 (1998)
National Cotton Council, Memphis TN

## The Optimization Model

Contemporary studies have found that both fertilizer applications and residual fertility have impacts on crop yields (Segarra, et. al.; Carter, Jensen, and Bosman). This manuscript addresses the impacts of phosphorus fertilizer application and residual on cotton yields under different levels of the soil fertility. That is, a dynamic optimization model is developed to evaluate the relationship between phosphorus application optimal decision rules and residual phosphorus. The model can be expressed as follows. Cotton yield is a function of phosphorus fertilizer application and phosphorus residual. Phosphorus residual at a given time is a function of the previous phosphorus applications and previous levels of phosphorus residual. Given these relationships, the general form of the optimization model is:

$$\text{Max } Z = \sum_{t=0}^{n} \{[\, P_t \cdot Y_t(PA_t, PR_t) - CP_t \cdot PA_t \,]\cdot(1+r)^{-t}\} \quad (1)$$

Subject to:

$$PR_{t+1} = f_t\,[PA_t\,,\, PR_t\,]\,, \quad (2)$$

$$PR_0 = PR(0)\,, \quad (3)$$

and $PA_t\,,\, PR_t \geq 0$ for all t.

Where, Z is the per-acre net present value of returns to risk, management, overhead, and all other inputs in the production of cotton ($/acre); n is the length of the decision-maker's planning horizon (years); $P_t$ is the price of cotton in year t ($/lb.); $Y_t$ is the cotton yield function in year t (lbs./acre); $PA_t$ is phosphorus applied in year t (lbs./acre); $PR_t$ is phosphorus residual in year t (lbs./acre); $CP_t$ is the price of phosphorus in year t ($/lb.); and r is the discount rate.

Equation (1) represents the objective function, or performance measure, of the optimization model. Equation (2) is the equation of motion, which updates phosphorus residual and is used as one of the variables in equation (1) to compute contemporary cotton yield. Equation (3) is the initial condition on the level of phosphorus residual at the beginning of the planning horizon.

The field experiments used to derive key relationships in the formulation of this model were conducted at the Agricultural Complex for Advanced Research and Extension Systems (AG-CARES) farm in Lamesa, Texas in 1995 and 1996. In 1995, five phosphorus fertilizer rates (0, 20, 30, 40 and 50) were applied to eight plots - each with 40 inch rows and 50 feet long. Eighteen replications of the plots were arranged in a randomized block design and located across areas of known variation in cotton yields. All input levels, other than phosphorus, were held at the same levels across the replications in the experiment. Cotton yields were calculated by obtaining hand-harvested cotton lint from each plot. Phosphorus residual in the top twelve inches of the soil in each block was measured in each of the two years. The phosphorus residual function, equation (2), was estimated using the data from both years.

Given an interest on evaluating the economic significance of optimal phosphorus use under non-homogeneous fertility of fields, the dynamic optimization model formulated above was solved under two scenarios. The first scenario used the 1995 experimental data described above to estimate an aggregate cotton yield function across all phosphorus residual levels. The second scenario used separate cotton yield functions according to three levels of phosphorus residual. This was done to mimic possible scenarios of fertility that could be faced under precision farming practices within fields. That is, under precision farming practices optimal input decision rules according to spatial differences within fields would be desired. In order to estimate the influence of different levels of phosphorus residual in the soil on phosphorus application and profit, the data set was divided into three groups, according to the 1995 total phosphorus residual (PR). Group 1 contained 29 observations in which PR ranged from 8.00 to 17.99 pounds per acre. Group 2 contained 29 observations in which PR ranged from 18.00 to 22.99 pounds per acre. Group 3 contained 30 observations in which PR ranged from 23.00 to 53.99 pounds per acre.

Using GLM procedures (SAS), the following functions were found to best represent the relationship between cotton yield and phosphorus application and residual in this experiment. Many functional forms including logarithmic, Mitscherlich-Spillman, quadratic, and cubic were tried. Overall fit of the data was not very good, the functional form found to fit the data best was the cubic form. Even though the estimated equations were not as statistically significant as desired, it was felt that analysis of these could provide insight into the economics of differentiating cotton yield response functions according to levels of phosphorus residual. Equation (4) gives the relationship, which used all the phosphorus residual levels. Equations (5), (6) and (7) present the relationships for Groups (1), (2) and (3), respectively.

Total: $$Y_t = 863.56 - 4.61PA_t + 11.38PR_t + 3.13*10^{-1}PA_t^2 - 3.71*10^{-1}PR_t^2$$
(5.03) (-0.70) (0.56) (0.95) (-0.48)
$$+1.05*10^{-1}PA_t*PR_t - 4.93*10^{-3}PA_t^3 + 2.35*10^{-3}Pr_t^3 \quad (4)$$
(0.99) (-1.15) (0.26)
$R^2 = 0.1143$

Group 1: $$Y_t = -1117.20 - 3.26PA_t + 552.44PR_t - 2.65*10^{-1}PA_t^2 - 46.90PR_t^2$$
(-0.40) (-0.21) (0.81) (-0.38) (-0.88)
$$+8.58*10^{-1}PA_t*PR_t + 2.70*10^{-3}PA_t^3 + 1.26PR_t^3 \quad (5)$$
(1.37) (0.29) (0.93)
$R^2 = 0.2167$

Group 2: $$Y_t = -12744.05 - 3.10*10^{-1}PA_t + 2104.35PR_t - 1.55*10^{-1}PA_t^2$$
(-0.13) (-0.02) (0.14) (-0.20)
$$-107.74PR_t^2 + 3.05*10^{-1}PA_t*PR_t + 1.27*10^{-3}PA_t^3 + 1.84Pr_t^3 \quad (6)$$
(-0.15) (0.38) (0.12) (0.15)
$R^2 = 0.1433$

Group 3: $$Y_t = 505.22 - 42.98PA_t + 61.84PR_t + 1.44PA_t^2 - 1.81PR_t^2$$
(0.28) (-3.02) (0.40) (2.85) (-0.42)
$$+5.50*10^{-1}PA_t*PR_t - 1.82*10^{-2}PA_t^3 + 1.11*10^{-2}PR_t^3 \quad (7)$$
(2.16) (-2.87) (0.29)

$R^2 = 0.3920.$

Where, $Y_t$, $PA_t$ and $PR_t$ are defined as previously stated. The values in parenthesis below the estimated parameters in each equation are the associated *t*-values.

Based on the information of phosphorus residual in the soil in 1995 and 1996, and phosphorus application in 1995, the phosphorus carry-over function was estimated to be:

$$PR_{t+1} = 7.63 + 1.98*10^{-1}PA_t + 4.08*10^{-1}PR_t \quad (8)$$
(2.28) (3.10) (2.91)
$R^2 = 0.2202.$

Where the variables are defined as before and the parameter *t*-values are reported as before. All the estimated parameters in equation (8) were significant at the 0.05 level.

## Results

The optimization model depicted in equations (1)-(3) was solved for all the combinations resulting from: (1) five alternative levels of cotton price (0.40, 0.45, 0.50, 0.55, and 0.60 \$/lb.); (2) five alternative levels of phosphorus fertilizer price (0.10, 0.15, 0.20, 0.25, and 0.30 \$/lb.); (3) a 5% discount rate (r = 0.05); and (4) a ten-year planning horizon. The initial conditions of phosphorus residual for the different groups were held at the average phosphorus residual of each group in 1995.

As expected, the optimal decision rules for applied phosphorus fertilizer varied across periods in the planning horizon for a given phosphorus and cotton price combination at the different levels of phosphorus residual. However, because a more stable optimal decision rule was desired to simplify management implementation, for a given phosphorus and cotton price combination and initial soil fertility, an additional constraint equating phosphorus applications across time periods within the planning horizon was introduced.

Solutions to the 100 optimization models (corresponding to four dynamic models, five cotton prices and five phosphorus prices) were obtained using GAMS (General Algebraic Mathematical System) and are presented in Tables 1 to 4. The top portion of each table depicts the optimal levels of phosphorus applications for the alternative cotton-phosphorus price combinations. The bottom portion of the tables depicts the associated net per-acre present value of returns.

As depicted in Table 1 to 4, the optimization models solve for specific, discrete combinations of phosphorus and cotton prices. However, such strict price combinations are unlikely to exist. Therefore, it was recognized that a generalized relationship based on relative prices of phosphorus-to-cotton rather than absolute prices would be more useful. Consequently, a generalization of the optimal phosphorus application decision rule was derived for each model. The procedure used was to regress the optimal phosphorus application against the phosphorus-to-cotton price ratio. For each dynamic model, the 25 optimal decision rules of phosphorus application were listed along with their associated phosphorus-to-cotton price ratios; five of these were eliminated since five alternative phosphorus-cotton price combinations for which the optimization model was solved had the same price ratios and thus the same optimal decision rule. The following functional form was then fitted to the remaining 20 points of the optimal decision rules for phosphorus applications and phosphorus-to-cotton price ratios:

$$e^{PA} = A * R^{\beta} * \varepsilon. \qquad (9)$$

Where "e" is the mathematical constant whose natural logarithm is equal to one; R is the phosphorus-to-cotton price ratio; PA is the optimal level of applied phosphorus; A and $\beta$ are the parameters to be estimated; and $\varepsilon$ is the error term. Regression results from the linearized form of equation (9) for the four models were:

Total: $$PA = \underset{(931.61)}{37.87} - \underset{(-19.95)}{0.76} \ln(R) \qquad (10)$$
$R^2 = 0.9544$

Group 1: $$PA = \underset{(228.74)}{19.69} - \underset{(-25.93)}{2.09} \ln(R) \qquad (11)$$
$R^2 = 0.9725$

Group 2: $$PA = 39.21 \qquad (12)$$
(It is a straight line.)

Group 3: $$PA = \underset{(3028.53)}{41.11} - \underset{(-19.48)}{0.25} \ln(R) \qquad (13)$$
$R^2 = 0.9523$.

Where the variables are defined as above and the values in parenthesis below the estimated parameters represent their associated *t*-values. All parameter estimates were significant at the 0.01 level. It is important to stress the fact that equations (10)-(13) were estimated to find an approximation of the continuous form of the phosphorus fertilizer optimal decision rules rather than to test their significance.

Equations (10)-(13) are presented graphically in Figure 1. As shown in this figure, when phosphorus-to-cotton price ratio is from 0.01 to 1.00, the optimal level of applied phosphorus fertilizer in pounds on a per acre bases ranges from 41.36 to 37.87 for Total; from 29.36 to 19.69 for Group 1; from 42.25 to 41.11 for Group 3; and it is constant at 39.11 for Group 2.

As expected, given the initial condition on phosphorus residual, the higher the phosphorus-to-cotton price ratio, the lower the optimal level of applied phosphorus fertilizer; and the lower the phosphorus-to-cotton price ratio, the higher the optimal level of applied phosphorus fertilizer. The only exception is Group 2, for which the optimal decision rule is quite stable. That is, for Group 2 the phosphorus-to-cotton price ratio does not influences the optimal level of applied phosphorus fertilizer.

As shown in Figure 1, there are not great differences among the optimal levels of phosphorus application for the Total, Group 2 and Group 3 optimal decision rules. But the optimal level of phosphorus application for Group 1 is significantly different from the solutions for other groups. Also, the solution for Group 1 is not as stable as the solutions for Group 2 and Group 3.

When comparing the results of the three groups, it can be seen that the higher the phosphorus residual, the higher the optimal level of applied phosphorus fertilizer; and that the lower the phosphorus residual, the lower the optimal level of applied phosphorus fertilizer. This result would seem to be counter-intuitive at first glance. However, this result implies that if particular sections of the field have low phosphorus residual levels, optimal target yields are likely to be low and it would not be as profitable to try to increase cotton yields in these portions of the field. Furthermore, this result also implies that given the 10 year planning horizon assumption, it would not be profitable to build up phosphorus residual in the soil when it is relatively low at the beginning of the planning horizon.

## Conclusion and Discussion

The objective of this manuscript was to evaluate the economic implications of precision farming practices, i.e., to derive phosphorus fertilizer application optimal decision rules under different soil fertility scenarios, which consider the dynamic phosphorus residual impacts of phosphorus application in irrigated cotton production. It was shown that given different soil fertility levels with respect to phosphorus, there exist differences on the optimal decision rule of phosphorus application. The difference is especially great when the soil fertility with respect to phosphorus is low, and the difference is relatively small when the soil fertility with respect to phosphorus is high.

## Acknowledgements

The authors acknowledge the contributions of Sukant Misra, Darren Hudson, and Blake Bennett to this manuscript. Department of Agricultural and Applied Economics, Texas Tech University, Cotton Economics Research Report CER-98-26.

## References

Segarra, E., D. E. Ethridge, C. R. Deussen, and A. B. Onken. 1989. "Nitrogen Carry-over Impacts in Irrigated Cotton Production, Southern High Plains of Texas." Western Journal of Agricultural Economics, 14(2): 300-309.

Carter, J. N., M. E. Jensen, and S. M. Bosman. 1974. "Determining Nitrogen Fertilization Needs for Sugarbeets From Residual Soil Nitrate and Mineralizable Nitrogen." *Agronomy J.* 66(1): 319-23.


Table 1. Per Acre Dynamic Optimal Levels of Applied Phosphorus and Associated Net Present Value of Returns for Alternative Cotton-Phosphorus Prices Across all Phosphorus Residual Levels, Assuming 20.96 lbs./acre Initial Condition on Phosphorus

| Phosphorus Price ($/lb.) | Cotton Price ($/lb.) 0.40 | 0.45 | 0.50 | 0.55 | 0.60 |
|---|---|---|---|---|---|
| | **Phosphorus Applied (lbs./acre/year)** | | | | |
| ***0.30*** | 37.91 | 38.08 | 38.22 | 38.34 | 38.43 |
| ***0.25*** | 38.17 | 38.31 | 38.43 | 38.52 | 38.60 |
| ***0.20*** | 38.43 | 38.54 | 38.63 | 38.71 | 38.77 |
| ***0.15*** | 38.68 | 38.77 | 38.83 | 38.89 | 38.93 |
| ***0.10*** | 38.93 | 38.99 | 39.03 | 39.07 | 39.10 |
| | **Net Present Value of Returns ($/acre, 10-year planning horizon)** | | | | |
| ***0.30*** | 3180 | 3589 | 3997 | 4406 | 4814 |
| ***0.25*** | 3195 | 3603 | 4012 | 4421 | 4829 |
| ***0.20*** | 3210 | 3618 | 4027 | 4436 | 4844 |
| ***0.15*** | 3225 | 3633 | 4042 | 4451 | 4859 |
| ***0.10*** | 3240 | 3648 | 4057 | 4466 | 4874 |

Table 2. Per Acre Dynamic Optimal Levels of Applied Phosphorus and Associated Net Present Value of Returns for Alternative Cotton-Phosphorus Prices in Group 1, Assuming 13.26 lbs./acre Initial Condition on Phosphorus

| Phosphorus Price ($/lb.) | Cotton Price ($/lb.) 0.40 | 0.45 | 0.50 | 0.55 | 0.60 |
|---|---|---|---|---|---|
| | **Phosphorus Applied (lbs./acre/year)** | | | | |
| ***0.30*** | 19.94 | 20.35 | 20.68 | 20.97 | 21.21 |
| ***0.25*** | 20.56 | 20.92 | 21.21 | 21.45 | 21.66 |
| ***0.20*** | 21.21 | 21.51 | 21.76 | 21.96 | 22.14 |
| ***0.15*** | 21.90 | 22.14 | 22.33 | 22.49 | 22.63 |
| ***0.10*** | 22.63 | 22.80 | 22.94 | 23.06 | 23.15 |
| | **Net Present Value of Returns ($/acre, 10-year planning horizon)** | | | | |
| ***0.30*** | 2989 | 3369 | 3748 | 4128 | 4508 |
| ***0.25*** | 2997 | 3377 | 3756 | 4136 | 4516 |
| ***0.20*** | 3005 | 3385 | 3765 | 4145 | 4524 |
| ***0.15*** | 3013 | 3393 | 3773 | 4153 | 4533 |
| ***0.10*** | 3022 | 3402 | 3782 | 4162 | 4542 |

Table 3. Per Acre Dynamic Optimal Levels of Applied Phosphorus and Associated Net Present Value of Returns for Alternative Cotton-Phosphorus Prices in Group 2, Assuming 20.34 lbs./acre Initial Condition on Phosphorus

| Phosphorus Price ($/lb.) | Cotton Price ($/lb.) 0.40 | 0.45 | 0.50 | 0.55 | 0.60 |
|---|---|---|---|---|---|
| | **Phosphorus Applied (lbs./acre/year)** | | | | |
| ***0.30*** | 39.21 | 39.21 | 39.21 | 39.21 | 39.21 |
| ***0.25*** | 39.21 | 39.21 | 39.21 | 39.21 | 39.21 |
| ***0.20*** | 39.21 | 39.21 | 39.21 | 39.21 | 39.21 |
| ***0.15*** | 39.21 | 39.21 | 39.21 | 39.21 | 39.21 |
| ***0.10*** | 39.21 | 39.21 | 39.21 | 39.21 | 39.21 |
| | **Net Present Value of Returns ($/acre, 10-year planning horizon)** | | | | |
| ***0.30*** | 3549 | 4004 | 4459 | 4914 | 5369 |
| ***0.25*** | 3564 | 4019 | 4474 | 4929 | 5384 |
| ***0.20*** | 3580 | 4035 | 4490 | 4945 | 5400 |
| ***0.15*** | 3595 | 4050 | 4505 | 4960 | 5415 |
| ***0.10*** | 3610 | 4065 | 4520 | 4975 | 5430 |

Table 4. Per Acre Dynamic Optimal Levels of Applied Phosphorus and Associated Net Present Value of Returns for Alternative Cotton-Phosphorus Prices in Group 3, Assuming 28.95 lbs./acre Initial Condition on Phosphorus

| Phosphorus Price ($/lb.) | Cotton Price ($/lb.) 0.40 | 0.45 | 0.50 | 0.55 | 0.60 |
|---|---|---|---|---|---|
| | **Phosphorus Applied (lbs./acre/year)** | | | | |
| ***0.30*** | 41.12 | 41.18 | 41.23 | 41.26 | 41.29 |
| ***0.25*** | 41.21 | 41.26 | 41.29 | 41.32 | 41.35 |
| ***0.20*** | 41.29 | 41.33 | 41.36 | 41.38 | 41.40 |
| ***0.15*** | 41.38 | 41.40 | 41.43 | 41.44 | 41.46 |
| ***0.10*** | 41.46 | 41.48 | 41.49 | 41.50 | 41.51 |
| | **Net Present Value of Returns ($/acre, 10-year planning horizon)** | | | | |
| ***0.30*** | 3251 | 3669 | 4087 | 4505 | 4924 |
| ***0.25*** | 3266 | 3685 | 4103 | 4521 | 4940 |
| ***0.20*** | 3282 | 3701 | 4119 | 4537 | 4956 |
| ***0.15*** | 3298 | 3717 | 4135 | 4553 | 4972 |
| ***0.10*** | 3314 | 3733 | 4151 | 4569 | 4988 |

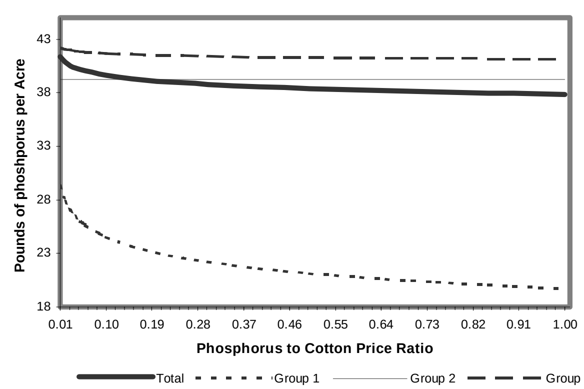


Figure 1. Continuous form of the optimal decision rule of applied phosphorus for different levels of phosphorus residual.