SPINNING QUALITY/PROCESS IMPROVEMENT THROUGH VARIANCE TOLERANCING

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Abstract

The means and variances for yarn tensile strengths were derived for ring card (RSK), combed (RSC) and open-end (OE) spun yarns of varying sizes as a direct function of the constituent fiber length distribution, single fiber tensile properties and the "effective gauge length," *L*. A large amount of test data from a 3-year production experiment was collected in order to test the concept and validate the theoretical models. The effective gauge length *L* was estimated for each yarn studied and the estimated yarn tensile strengths were compared against the actual yarn strengths. By applying the "variance tolerancing" techniques, the total variance of the actual yarn tensile strength was decomposed into the between-package variance (24%) resulting from spinning processes and the within-package variance (76%). The latter, in turn, was decomposed into random component of the fiber properties (18%) and the nonrandom component (58%) that was process dependent. Effects of fiber length and strength on the resulting yarn tensile strength were also examined theoretically.

Introduction

The large textile science and engineering knowledge base has not been much utilized for textile production and quality control for reasons that are well explained by Suh (1). Observing this as a failure in textile research, he has considered several mechanisms of failure at work, and suggested new directions for the future.

Predicting quality characteristics of textile structures from the input variables has been the target of many research attempts in the past. The prediction equations proposed by most of these can be categorized as either statistical (2-4) or mechanistic (5-7).

The studies based on statistical approaches (2-4) often have used regression and correlation analyses in place of finding the true underlying structural relationships. The estimated coefficients thus found from specific populations and operating conditions are often highly volatile and unstable, proving that a statistically significant relationship does not necessarily guarantee the existence of a true cause-effect relationship.

Multifactor, multi-variate models (5-7) based on deterministic, non-stochastic models have long been used to depict the "average phenomena" (signal) based on the input or predictor variables. Depending on the functional forms, the variance and/or the coefficient of variation (CV) of the output factor is often found to be much greater than that of the individual input factors. As the complexity of the functional form and the number of predictor variables increases, the precision of the factor to be predicted quickly diminishes. This is due to the nonuniformity and inconsistency of the response pattern over the entire ranges of the predictor variables that are often highly correlated among themselves. When this reality is added to the variance (noise) introduced by each process (process variance), it is not surprising at all that the multitudes of the existing forward prediction equations are seldom used by the practitioners of quality control and improvement in textile manufacturing.

For these reasons, the traditional research results have been largely ineffective in improving textile product/process qualities. The future control and improvement strategies of textile process qualities, therefore, must come from a proper analysis of the variance on input and output variables along with a valid structural relationship. The following is an attempt for this in predicting tensile strengths of spun yarns.

Variance Tolerancing and Decomposition from Fiber to Yarn

In textile manufacturing operations, the processing can provide good estimates for variance components resulting from raw materials, processing conditions and time. In a spun yarn or a woven fabric, the variance of each quality characteristic can be decomposed into that of sub-components under certain structural relationships. The tensile strength is one of the most critical quality characteristics of a spun yarn as it determines the ends-down rate during spinning and loom stops during weaving which lead to quality and profit losses.

The total variance of tensile strength (σ^2_{yts}) for an actual yarn can be decomposed into between-package variance (σ^2_{bp}) and within-package variance (σ^2_{wp}). The between-package variance is entirely due to variations among the processing machines accrued at different stages of spinning. The within-package variance is further decomposed into random (σ^2_{r}) and nonrandom (σ^2_{nr}) sub-components. The variance from a random component is the variance arising from the fiber properties and the randomness associated with the arrangements of the constituent fibers. The variance from nonrandom component within a machine refers to the variation caused by the systematic fluctuations of fiber mass due to drafting waves and long-term drifts of machine effects. In order to decompose the within-package variance, the variance due to random component must be estimated through "variance tolerancing" (8).

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Theoretical Aspects

Let the yarn under consideration be a chain of extended bundles of fibers which are aligned parallel to the length axis of the bundle with effective gauge length L, as shown in Figure 1. According to the weakest-link theory of F.T. Peirce (9), the strength of a yarn may be modeled and estimated by the strength of the weakest bundle within a chain of bundles. In turn, the weakest bundle can be assumed to be the one which contains the smallest number of continuous fibers with an arbitrary interval size "L" within a yarn, when there are r such bundles within a test length rL. While the underlying theories were already given in the previous paper (10), the following summarizes the equations applied to this study.

Let

Y_1 = the smallest number (among "r") of continuous fibers within L,

E[Y_1], Var[Y_1] = expectation and variance of Y_1,

FS = breaking strength of a single fiber,

E[FS], *Var*[FS] = expectation and variance of single fiber strength, respectively,

ϵ_{BS} = bundle strength efficiency,

YTS = tensile strength of a spun yarn, which is equivalent to

$$\sum_{i=1}^{Y_1} \mathrm{Fs}_i \cdot \epsilon_{BS},$$

E[YTS], *Var*[YTS] = expectation and variance of yarn tensile strength, respectively.

Under the structural geometry of a spun yarn, *E*(YTS), *Var*(YTS) can be estimated by using a variance tolerancing technique. The smallest number of continuous fibers for 14.25/1 Ne, 20/1 Ne and 40/1 Ne spun yarns are given in Tables 1 ~ 3. The same for 6.3/1 Ne spun yarn was given in the previous paper (10).

$$E[\mathrm{YTS}] = \underset{Y_1}{E}\,\underset{YTS}{E}\left[YTS/Y_1\right] = \underset{Y_1}{E}\left[\sum_{i=1}^{Y_1} FS_i \cdot \epsilon_{BS}\right],$$

$$= \epsilon_{BS} \cdot E[\mathrm{FS}] \cdot E[Y_1], \quad (1)$$

$$Var[\mathrm{YTS}] = \underset{Y_1}{E}\,\underset{YTS}{Var}\left[\mathrm{YTS}/\mathrm{Y}_1\right] + \underset{Y_1}{Var}\,\underset{YTS}{E}\left[\mathrm{YTS}/\mathrm{Y}_1\right],$$

$$= \underset{Y_1}{E}\,\underset{YTS}{Var}\left[\sum_{i=1}^{Y_1} FS_i \cdot \epsilon_{BS}\right] + \underset{Y_1}{Var}\,\underset{YTS}{E}\left[\sum_{i=1}^{Y_1} FS_i \cdot \epsilon_{BS}\right],$$

$$= \epsilon_{BS}^2 \cdot Var[FS] \cdot E[Y_1] + \{E[FS]\}^2 \cdot Var[Y_1]. \quad (2)$$

Based on the Equations (1) and (2), the expectations and variances of yarn tensile strength for 20/1 Ne RSK, 20/1 and 40/1 Ne RSC and 6.3/1, 14.25/1 and 20/1 Ne OE spun yarns were calculated with a fixed value of bundle strength efficiency, ϵ_{BS}. The particular value of ϵ_{BS} was determined by the range of Y_1, the smallest number of fibers within L. When the size of a bundle is between 150 and 500, the bundle strength efficiency was reported to be 0.58 based on an earlier study by Cui (11). The MANTIS® single fiber test results were used for obtaining the single fiber tensile properties.

Effective Gauge Length

For each yarn sample, the effective gauge length L was so determined that the expected values of yarn tensile strengths came close (within around 1.6%) to the actual average tensile strengths obtained from a TENSORAPID®.

Table 4 shows the effective gauge length L for six different yarns. All six yarns were produced with an identical twist multiplier of 4.5. In RSC yarns, the effective gauge length L was found to be smaller for finer yarns. It, however, did not appear to change much for OE yarns where the fibers are less parallel to each other compared to RSC yarns.

For the 20/1 Ne yarns, the effective gauge length was found to be the smallest for the RSC yarn, followed by RSK and OE yarns. This might have been due to the structural differences among the ring and open-end spun yarns.

The Effects of Fiber Length and Strength on Yarn Tensile Strengths

Based on the magnitudes of "toleranced variance," it was found that the variance of yarn tensile strength inherent to fiber properties is determined largely by the mean and variance of the smallest number of continuous fibers within L as well as the strengths of the constituent fibers. On the other hand, the smallest number of continuous fibers within L depends on the fiber length distribution.

The effects of mean fiber length, mean fiber strength and standard deviation of fiber length on tensile strengths of 14.25/1 Ne OE, 20/1 Ne RSC and 20/1 Ne RSK yarns are shown in Figures 2 ~ 4.

Figure 2 shows three response surfaces obtained for three different standard deviations of fiber lengths (0.40, 0.45 and 0.50 inch). It is seen that the yarn tensile strength (EYTS) increases as the fiber length (FL) and fiber strength (FS) increase.

The height of the surface is shown to decrease as the standard deviation of fiber length increases for each of the three yarns in Figures 2 ~ 4. It is also shown that the yarn tensile strength decreases as the standard deviation of fiber length increases. The decrease becomes much smaller for the finer yarns, implying that the effects of fiber length variation on yarn tensile strength is smaller for finer yarns.

Effects of CV% of fiber strength and of the length are compared against that of yarn tensile strengths are shown in Figures 5 ~ 7.

Figure 5 shows two response surfaces formed by two different values of effective gauge length *L*. The CV% of yarn tensile strength is shown to increase as the effective gauge length increases. The CV% of fiber strength shows a significant positive effect on the CV% of yarn tensile strength. The CV% of fiber length is also shown to influence the CV% of yarn tensile strength, but to a much lesser extent.

Effects of fiber length and fiber strength on tensile strengths of other yarns are similar and thus not shown here.

Variance Decomposition of Yarn Tensile Strength

The variance of the actual yarn tensile strength (σ^2_{yts}) can be decomposed into the between-package variance (σ^2_{bp}) and within-package variance (σ^2_{wp}). These two components were estimated from the actual test data using the SAS® VARCOMP procedure. The within-package variance (σ^2_{wp}) is again decomposed into random (σ^2_{r}) and nonrandom (σ^2_{nr}) sub-components.

The variance of yarn tensile strength due to random component was obtained from the theories shown in the section A. The decomposed variance are given in Table 5. Figures 8 ~ 10 show the decomposed variances of the actual yarn tensile strengths for the three selected yarns. Instead of the variances, CV% were shown in order to compare the magnitudes of the variation directly for all yarn counts.

Figures 11 ~ 12 show the effects of yarn count on the CV% of yarn tensile strength due to random components (CVr%) and due to nonrandom components (CVnr%) for the OE and RSC yarns studied. For both yarns, CVr% and CVnr% are shown to increase as the yarns become finer.

Figure 13 shows the effects of spinning processes on the CV% of yarn tensile strength due to random components (CVr%) and due to nonrandom component (CVnr%) for 20/1 Ne OE, RSK and RSC yarns.

The variances of yarn tensile strength due to random components (CVr%) were shown to be the highest for the OE spun yarns, followed by the RSK and RSC yarns. This suggests that the variance due to random components decreases as the parallelness of fibers improves.

The variances of yarn tensile strength due to nonrandom components (CVnr%) were the lowest for RSC yarns followed by OE and RSK yarns, owing to the fact that the RSC yarns were produced through a combing process, which improved the orientation and alignment of fibers in the sliver.

The variances of yarn tensile strength due to nonrandom components (CVnr%) are shown to be lower for the OE spun yarns than for RSK yarns. This implies that the nonrandom variance components are smaller for OE yarns. In open-end spinning, individual fibers are deposited in layers on the surface of rotor in the absence of irregularities introduced by the roller drafting in ring spinning.

Conclusions

Yarn tensile strength was modeled and estimated by combining the structural geometry of fibers within a yarn with a statistical model for explaining the strength of fiber bundles based on the fiber length, strength, fineness and the effective gauge length "*L*."

More specially, a spun yarn was considered to be a chain of twisted parallel bundles with a known distribution of the number of "continuous" fibers within an arbitrary interval within the yarn called effective gauge length (*L*).

The effective gauge length *L* could be determined for every yarn studied with less than 1.6% differences between the simulated and actual yarn tensile strengths. The effective gauge length *L* was shown to decrease in RSC yarns as the yarns became finer. It was, however, not affected by the yarn twists in OE yarns.

The exact means and variances were derived for the tensile strength of spun yarns as direct functions of the distributions of fiber length, single fiber strength, and the effective gauge length "*L*" specific to the constituting fibers. The results were based on six yarns (6.3/1, 14.25/1, and 20/1 Ne OE, 20/1 Ne RSK and 20/1 and 40/1 Ne RSC) and 70,000 MANTIS® single fiber tensile test data.

The variances of the actual yarn tensile strengths were successfully decomposed into two components; between-package variance (24%) and the within-package variance (76%). While the between-package variances were entirely due to the variations among the processing machines accrued at different stages of spinning, the within-package variances were due to the input variances of the random components(18%) coming from raw materials and the process variances generated within a machine (58%).

The effects of the fiber length and single fiber strength on yarn tensile strength were similar to each other for the six different yarns studied. The variance of fiber length was shown to have a large effect on the mean of the estimated yarn tensile strength, and its effect was smaller for finer yarns. The variance of fiber strength, on the other hand, had a large effect on the variance of the estimated yarn tensile strength.

The toleranced variance of yarn tensile strength from fiber properties were the highest for OE yarns (5.87 ~ 7.22%), followed by RSK (5.09 ~ 6.33%) and RSC (4.72 ~ 6.03%) yarns. These differences were thought to be due to the differences in the fiber arrangements resulting from the different spinning processes. The variance of yarn tensile strength due to nonrandom components were shown to be

lower for OE yarns (6.80 ~ 8.00%) than for RSK yarns (8.06 ~ 8.90%). This might have been due to the fact that drafting waves or long-term drifts of machine effects are less likely to exist in OE spinning system. The RSC yarns (6.26 ~ 7.30%) were shown to exhibit lower variances than RSK yarns (8.06 ~ 8.90%) perhaps due to the combing process which improved the uniformity of the resulting yarns.

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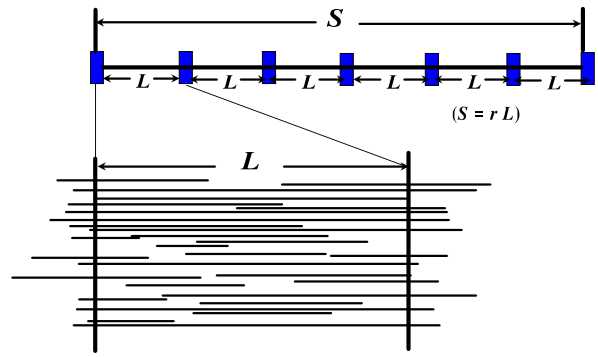

Figure 1. Model for Spun Yarn Strength.

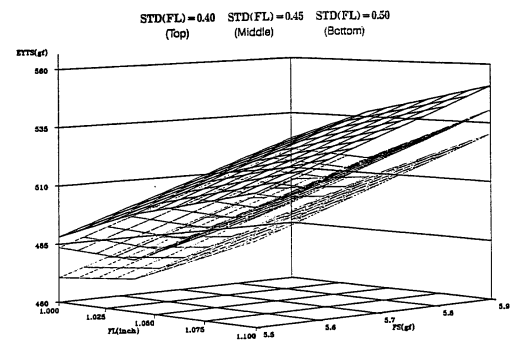

Figure 2. Fiber Length(FL) and Strength(FS) vs. Estimated Yarn Tensile Strength (EYTS) (14.25/1 Ne OE, *L*=0.30 inch).

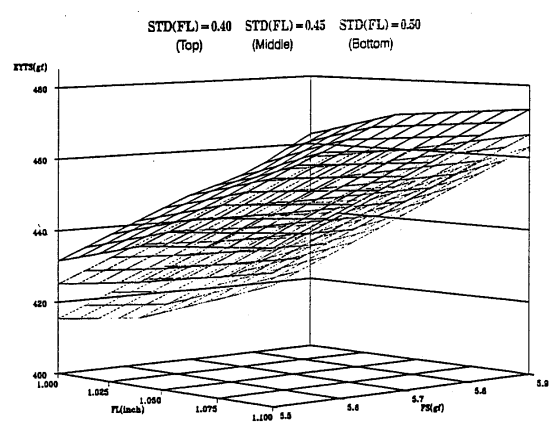

Figure 3. Fiber Length and Strength vs. Estimated Yarn Tensile Strength (20/1 Ne RSK, *L*=0.15 inch).

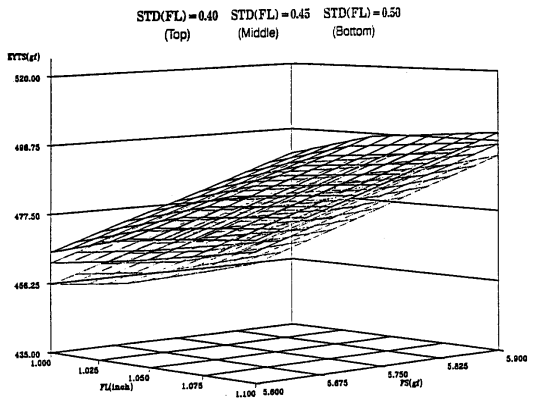

Figure 4. Fiber Length and Strength vs. Estimated Yarn Tensile Strength (20/1 Ne RSC, *L*=0.10 inch).

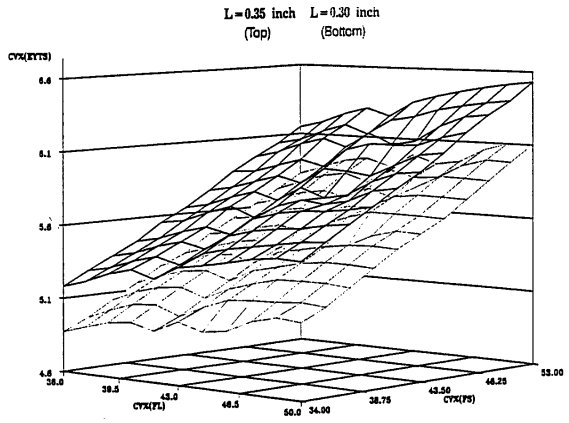

Figure 5. CV% of Fiber Length and CV% of Fiber Strength. CV% of Estimated Yarn Tensile Strength (14.25/1 Ne OE).

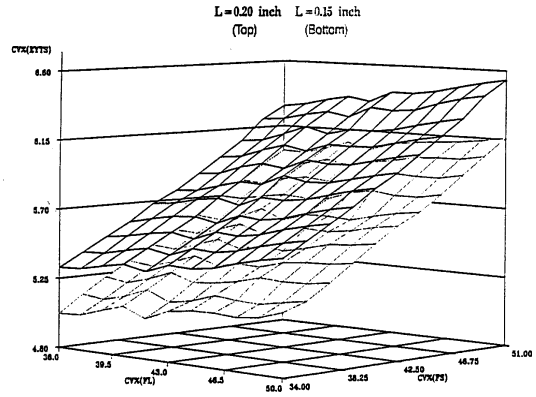

Figure 6. CV% of Fiber Length and CV% of Fiber Strength. CV% of Estimated Yarn Tensile Strength (20/1 Ne RSK).

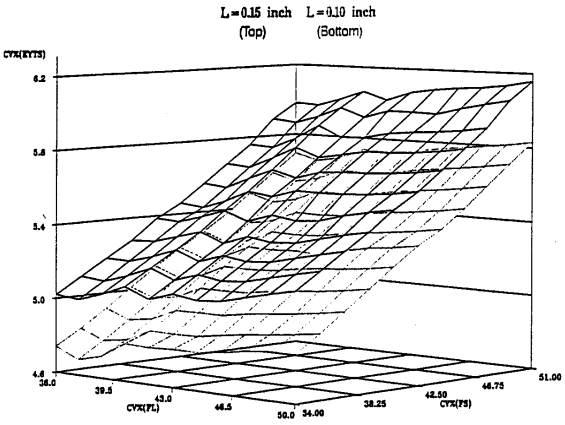

Figure 7. CV% of Fiber Length and CV% of Fiber Strength CV% of Estimated Yarn Tensile Strength (20/1 Ne RSC).

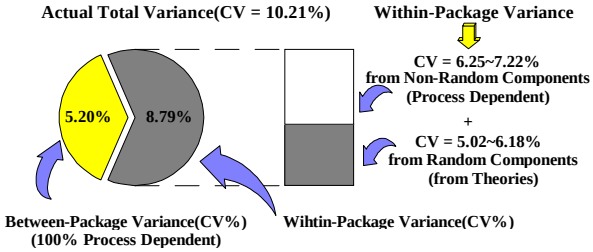

Figure 8. Variance Decomposition of 14.25/1 Ne OE Yarn Tensile Strength.

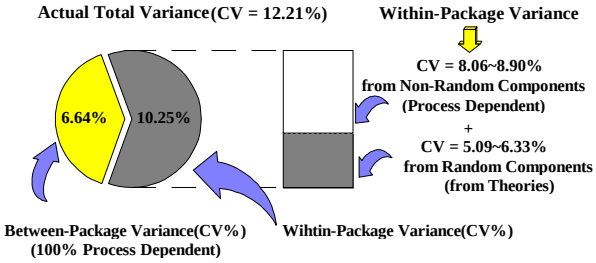

Figure 9. Variance Decomposition of 20/1 Ne RSK Yarn Tensile Strength.

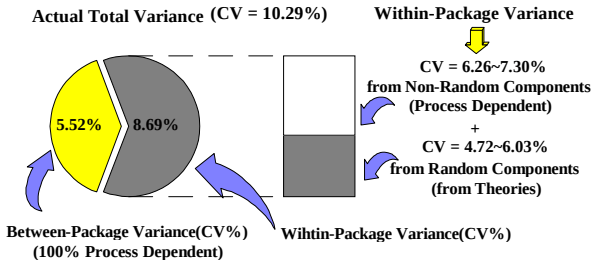

Figure 10. Variance Decomposition of 20/1 Ne RSC Yarn Tensile Strength.

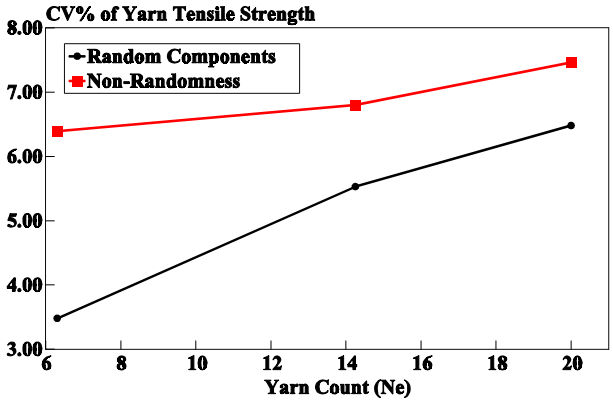

Figure 11. Effect of Yarn Count on the CV% of Yarn Tensile Strength (OE).

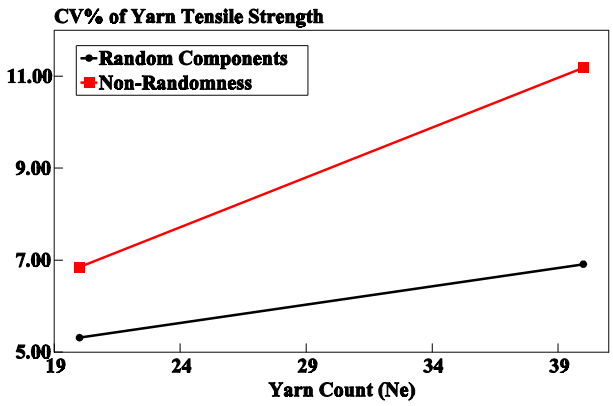

Figure 12. Effect of Yarn Count on the CV% of Yarn Tensile Strength (RSC).

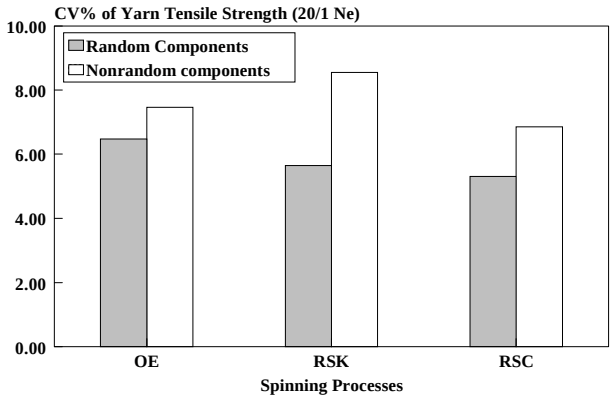

Figure 13. Effect of Spinning Processes on the CV% of Yarn Tensile Strength.

Table 1. The Smallest Number of Continuous Fibers within L (14.25/1 Ne)

Fiber Length (inch)		Effective Gauge Length *L* (inch)							
		0.25		0.30		0.35		0.40	
Mean	STD	Mean	Var	Mean	Var	Mean	Var	Mean	Var
1.00	0.40	168	42	153	41	139	39	125	36
	0.45	164	42	149	40	135	38	121	35
	0.50	159	40	145	38	130	36	117	34
1.04	0.40	171	43	157	42	144	40	130	38
	0.45	166	42	151	40	137	38	124	36
	0.50	161	41	146	39	132	37	118	34
1.10	0.40	174	44	161	43	147	41	135	39
	0.45	171	43	158	42	144	40	131	38
	0.50	169	43	155	41	141	39	128	40

Table 2. The Smallest Number of Continuous Fibers within *L* (20/1 Ne)

Fiber Length (inch)		Effective Gauge Length *L* (inch)									
		0.10		0.15		0.20		0.25		0.30	
Mean	STD	Mean	Var	Mean	Var	Mean	Var	Mean	Var	Mean	Var
1.00	0.40	141	29	133	31	123	29	113	29	103	28
	0.45	140	29	131	30	121	29	111	28	101	27
	0.50	138	29	128	29	118	29	108	28	98	26
1.04	0.40	143	28	135	30	125	30	116	30	107	29
	0.45	141	30	132	30	121	29	112	29	102	27
	0.50	139	29	129	29	119	29	109	27	99	27
1.10	0.40	144	30	136	31	127	31	118	30	109	29
	0.45	143	30	134	30	125	30	116	30	107	29
	0.50	142	29	133	30	123	30	113	29	104	28

Table 3. The Smallest Number of Continuous Fibers within *L* (40/1 Ne)

Fiber Length (inch)		Effective Gauge Length *L* (inch)							
		0.07		0.10		0.13		0.16	
Mean	STD	Mean	Var	Mean	Var	Mean	Var	Mean	Var
1.00	0.40	68	13	66	14	64	14	62	14
	0.45	68	13	65	14	64	14	61	14
	0.50	67	13	65	14	63	14	61	14
1.04	0.40	68	13	66	14	64	14	62	15
	0.45	68	13	65	14	64	14	62	14
	0.50	67	13	65	14	63	14	61	14
1.10	0.40	69	14	67	14	65	15	63	15
	0.45	68	13	66	14	64	14	62	15
	0.50	68	13	66	14	64	14	62	14

Table 4. Effective Gauge Length *L* for 6 Different Yarns

Yarn Count	Spinning Method	Effect Gauge Length *L* (inches)
6.3/1	OE	0.30 ~ 0.35
14.25/1	OE	0.30 ~ 0.35
20/1	OE	0.25 ~ 0.30
	RSK	0.15 ~ 0.20
	RSC	0.10 ~ 0.15
40/1	RSC	0.07 ~ 0.10

Table 5. Variance Decomposition of Yarn Tensile Strengths

Yarn Count	Spinning Method	CVbp (%)	CVwp (%)	*L* (inch)	CVr (%)	CVnr (%)
6.3/1	OE	4.06	7.28	0.30	3.17 ~ 3.81	6.20 ~ 6.55
				0.35	3.24 ~ 3.86	6.17 ~ 6.52
14.25/1	OE	5.20	8.79	0.30	5.02 ~ 6.05	6.38 ~ 7.22
				0.35	5.20 ~ 6.18	6.25 ~ 7.09
20/1	OE	7.35	9.92	0.25	5.87 ~ 7.11	6.92 ~ 8.00
				0.30	6.01 ~ 7.22	6.80 ~ 7.89
	RSK	6.64	10.25	0.15	5.09 ~ 6.25	8.12 ~ 8.90
				0.20	5.27 ~ 6.33	8.06 ~ 8.79
	RSC	5.52	8.69	0.10	4.72 ~ 5.89	6.39 ~ 7.30
				0.15	4.97 ~ 6.03	6.26 ~ 7.13
40/1	RSC	4.70	13.22	0.07	6.36 ~ 7.65	10.78~ 11.59
				0.01	6.59 ~ 7.78	10.69 ~ 11.46