SMALL BUNDLE TENSILE PROPERTIES OF COTTON RELATED TO MANTIS® AND HVI DATA - A ROAD TO YARN STRENGTH PREDICTION

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Abstract

It is a well established fact that the strength of a yarn and its variance are determined by the tensile properties of constituent fibers. However, the results from either single fiber or bundle tensile testing alone do not predict the yarn tensile properties well due to the fact that the bundle strength efficiencies and variances are functions of bundle size. A theory was developed to predict the strengths and variances of small fiber bundles in terms of the bundle size and the single fiber tensile properties. Cotton fibers were tested on a Mantis® single fiber tester, and the results were applied in validating the theoretical models developed. The tensile properties of small bundles were also obtained by superposition of single fiber load-elongation curves from MantisR tests.

The results from the model showed an excellent agreement with the results of computer simulation of bundle strength, its standard deviation and CV.

Introduction

Beginning from 1991, all U.S. cotton produced under the federal price support loan program has been required to be HVI tested. The test results are stored in a database managed by Cotton Incorporated and can be acquired electronically national wide. Results from HVI tests have been used for not only classing cottons, but also optimizing laydown formations and predicting yarn tensile properties. In relating HVI tensile test results to yarn properties, there are two important issues that have not been addressed so far: the variances of fiber tensile properties and the effects of the bundle size.

It should be realized that the variance of yarn strength is, to a certain extent, more important than its average. This is so because the textile processing efficiencies are largely determined by the variation of the yarn strength along the yarn axis rather than the average itself. It is the weak places that usually break during the manufacturing process. The variance of yarn strength is no doubt related to the variance of single fiber tensile properties. Unfortunately, the information on the variance of single fiber tensile properties cannot be easily obtained from HVI test results. Furthermore, the variance of yarn, as a small fiber bundle, cannot be properly estimated from HVI data directly because of the size.

In addition, the bundle strength, its variance, CV and strength efficiency change with the bundle size. Theoretical study of small bundle tensile properties has been carried out by Peirce [1], Danials [2], Coleman [3], and Suh [4, 5, 6]. Based on these studies, the bundle strength efficiency is known to decrease as the bundle size increases, and to reach its asymptotic value as the bundle size reaches an infinity. It will be shown in this paper that the variance of bundle breaking strength increases but its CV decreases as bundle size increases.

The bundle strength efficiency is defined as the ratio of the realized bundle strength to the sum of the strengths of the constituent fibers in the bundle.

A yarn cross section may contain several tens to hundreds of fibers depending on the yarn size and the fiber fineness. Only a portion of the fibers within the yarn cross section usually break while other fibers slip when the yarn breaks. The percentage of broken fibers was reported to be in the range of 33 - 52% [7]. On the other hand, about 1,500 to 2,000 fibers are broken in an HVI tensile test. Obviously, an in-depth study on small bundle properties is needed in order to make use of the HVI test data.

In this paper, tensile properties of bundles of varying sizes, more specifically, the effects of bundle sizes on the variance of bundle strengths and bundle strength efficiencies, are reported. The theoretical results in this paper were based on Suh's earlier work, while the experimental work was based on computer simulations as explained below.

Single fibers were tested by use of a single fiber tensile tester named Mantis® and manufactured by Zellweger Uster. More than 1,000 data points representing the entire load-elongation curve of each fiber were stored in the computer.

A computer program was developed for randomly selecting a specified number of single fiber load-elongation curves and also for superposing them to form bundle load-elongation curves. Each single fiber load-elongation curve was randomly chosen from the database containing over 20,000 single fiber load-elongation curves for each cotton type. Two methods of superposition were employed for each set of randomly chosen single fiber data. One method includes the use of fiber crimp as well as the elongation, and the other includes the elongation only.

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Theories

According to Suh's earlier work[4, 5, 6], for n fibers having strengths X1, X2, ..., Xn and their corresponding order statistics Y1, Y2, ..., Yn, the strength of an n-fiber bundle Bn is defined as

$$B_n = \max_{1 \le k \le n} \{(n-k+1)Y_k\}, \qquad 0 \le Y_1 \le Y_2 \le \ldots \le Y_n$$

The cumulative distribution function, Sn(x), for Bn is

$$S_n(x) = \sum_{k=1}^{n} (-1)^{k+1} \binom{n}{k} F[(x/n)]^k S_{n-k}(x), \; 0 \le x \qquad (1)$$

where F(x) is the cumulative distribution function for single fiber breaking strength, and

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

The expected strength, $E(B_n)$, of a bundle with n fibers and its variance, $V(B_n)$, can be calculated by using the equations:

$$E(B_n) = \int_0^\infty [1 - S_n(x)]dx, \qquad (2)$$

$$E(B^2_n) = \int_0^\infty x[1 - S_n(x)]dx, \qquad (3)$$

where $S_0(x) = 1$ and $S1(x) = F(x)$.

When the bundle size n is sufficiently large, $\frac{Bn}{n}$ is normally distributed,

$$\frac{Bn}{n} \sim A.N.\left([1 - F(x_0)]x_0, \frac{1}{n}F(x_0)[1 - F(x_0)]x_0^2\right). \qquad (4)$$

where $x[1 - F(x)]$ obtains maximum at $x = x_0$.

Results and Discussions

The seemingly simple equation for $Sn(x)$ turned out to be very difficult to calculate even with a powerful computer as the bundle size n increases. For example, the value of $\binom{60}{30}$ is 118264581564861424, while the actual value used by a 64 bits computer may be 1.1826458156486142 x 10^{16}, causing an error of 4 in the last digit. One should bear in mind that the value of $Sn(x)$ should be between 0 and 1 since it is a cumulative distribution function. Therefore, this seemingly small error would cause a significant amount error in the final result. The equation also demands a high accuracy for the accumulative distribution function of single fiber breaking strength, $F(x/n)$. Even if the value of $F(x/n)$ is accurate to the tenth digit, its product with a large number of $\binom{n}{k}$ may still produce an unacceptable error. Further, the equation for Sn(x) is highly recursive, making the computation unmanageable as n increase due to problems associated with overflows and loss of precision in the computing.

After many trials, the expected bundle strength and its variance could be calculated up to bundle sizes of about 50. The computing time for the average strength and variance of a 50-fiber bundle was reduced from several hours to about 20 minutes with a workstation (DECstation 5000/25).

Although a cumulative distribution function from single fiber test results can be used directly in calculating $Sn(x)$, the lack of smoothness of the function due to limited sample size may produce less desirable results. Experimental results and statistical analysis have shown that the cumulative distribution functions of cotton fiber breaking strengths can be approximated by Weibull distributions [8, 9]. Therefore, the parameters of Weibull distributions were estimated based on Mantis® test results in order to calculate $Sn(x)$.

Three U.S. Upland cottons, named "B Cotton," "I Cotton" and "T Cotton," were selected for this study. The computation results on these cottons are shown in Figures 1 through 3 for bundle strength efficiency, Figures 4 through 6 for the standard deviation of bundle breaking strength and Figures 7 through 9 for the CV% of bundle breaking strength.

The computer program developed was used to randomly choose n fiber load-elongation curves from the database containing about 20,000 curves for each of "B Cotton," "I Cotton" and "T Cotton." They were superposed to form bundle load-elongation curves. Two types of bundle load-elongation curves were generated, each with n single fiber load#elongation curves; one with fiber crimps (slack bundle) and the other without (parallel bundle). The bundle size n was varied from 2 through 2,000. A total of 500 simulations were performed for each bundle size for each cotton. The bundle strength efficiency, standard deviation and CV% of bundle breaking strength thus obtained for each cotton are shown in Figures 1 through 9.

It can be seen clearly from the results that bundle strength efficiency decreases but not linearly as the bundle size increases. This indicates that any model for predicting yarn strength should take the bundle size effects into consideration.

The standard deviation of bundle breaking strength is shown to increase as the bundle size increases, whereas, the CV% decreases as the bundle size increases.

It should be pointed out that each simulated bundle load#elongation curve was obtained by summing up the loads of the surviving fibers at each elongation point. Although Suh's early model was developed by applying the assumptions that the bundle load at any given bundle extension is shared equally by the surviving fibers at that extension and that all fibers are parallel to each other with equal lengths (i.e., classical bundle), it is clear from the simulation results that tensile properties of the slack fiber bundles match that of the classical bundles extremely well in every case.

The classical bundle theories also provided a way to estimate the strength and variance of large fiber bundles. Based on Equation 4, the variance and CV of bundle strength are

$$\sigma^2 = {}^1/_n\, F(x_o)[1 - F(x_o)]\, x_0^2 \quad \text{and} \tag{5}$$

$$CV = \sqrt{\frac{x_0 F(x_0)}{n[1 - F(x_o)]}} = \frac{k}{\sqrt{n}} \tag{6}$$

From the equation, it can be seen clearly that the variance and CV depend on the distribution of fiber breaking strength and the bundle size. Based on the single fiber test data of " B Cotton" from MantisR, the value of k was found to be 2.11. By applying this, the CV of bundle breaking strength for 1,600-fiber bundles turned out to be 5%. This value is equivalent to the variation inherent to the variations of the fiber tensile properties. The k value also provides guidelines for setting confidence limits for HVI strength testing.

Table 1. HVI sample size vs. acquired precision for strength*

No. of Tests	95% Confidence Limits
1	#2.94
2	#2.08
4	#1.47
8	#1.04
12	#0.85
16	#0.74
24	#0.60

* Based on mean = 30 grams/tex, n = 1,600 (bundle size) and SD = 1.5 (from Mantis data and asymptotic variance).

The strength model for small fiber bundles and the computational methods developed here provide a new powerful tool for studying the relationship between single fiber and bundle tensile properties. As a result, it is expected that the tensile properties of spun yarns can be predicted more effectively in the further.

In addition, the revealing of the relationships among single fiber, small bundle and large bundle tensile properties is also expected to facilitate a better interpretation and more effective utilization of the HVI test results.

Conclusions

1. Computer simulation using MantisR single fiber tensile data is an effective method for predicting the tensile properties of bundles of different sizes.

2. As the bundle size increases, the bundle strength efficiency and CV% of
bundle breaking strength decrease, whereas the standard deviation of the strength increases nonlinearly.

3. The average bundle breaking strength, its standard deviation and CV from the classical bundle theories match well the simulation results from single
fiber data.

4. The "classical bundle theory," combined with the MantisR single fiber data, suggests that the sample size for HVI strength tests should be greater than the current practice (n=1,2) in order to obtain an acceptable precision from the tests.

Acknowledgments

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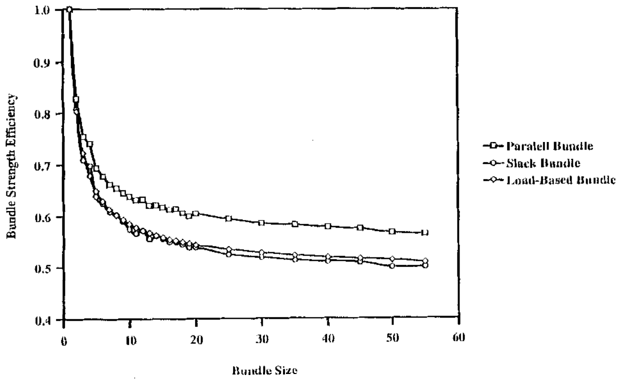

Figure 1. Bundle strength efficiency ("B Cotton")

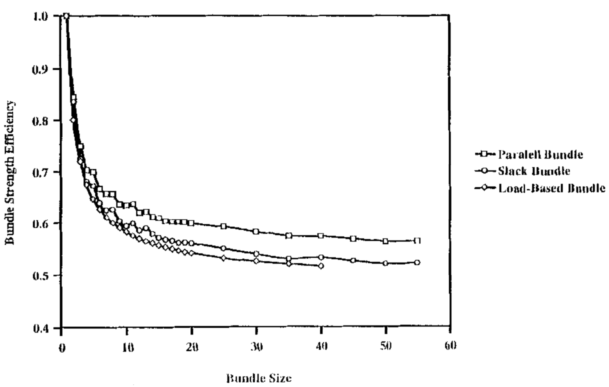

Figure 2. Bundle strength efficiency ("I Cotton")

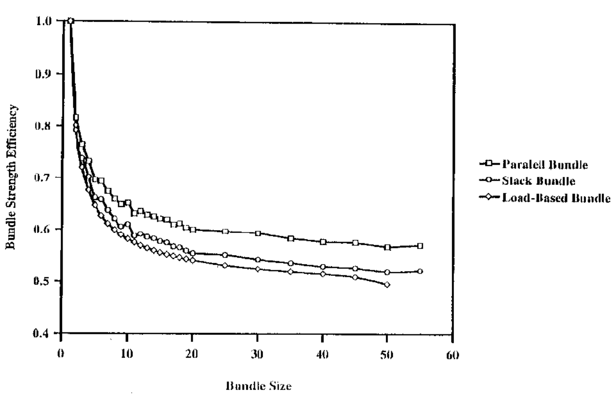

Figure 3. Bundle strength efficiency ("T Cotton")

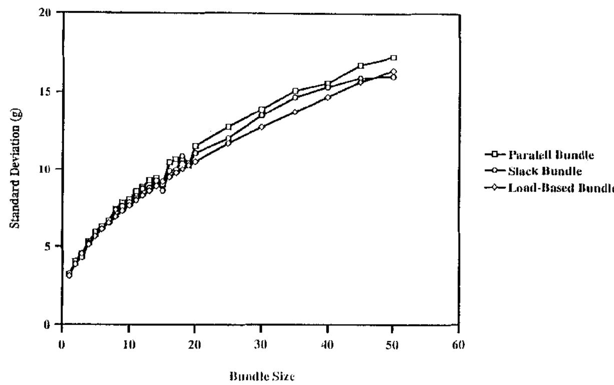

Figure 4. Standard deviation of bundle strength ("B Cotton")

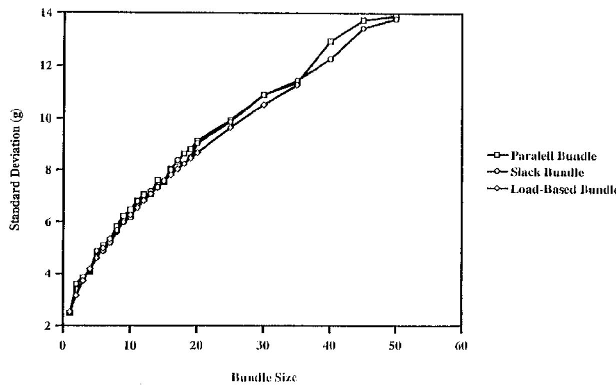

Figure 5. Standard deviation of bundle strength ("I Cotton")

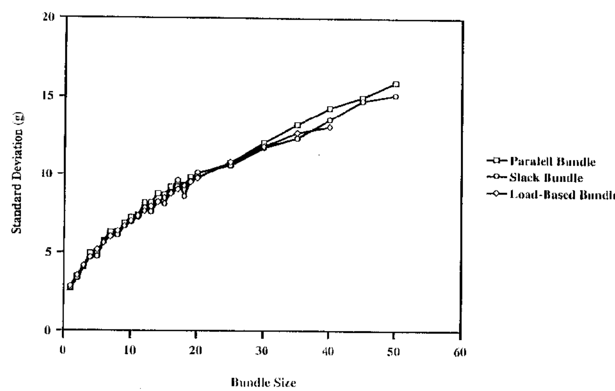

Figure 6. Standard deviation of bundle strenth ("T Cotton")

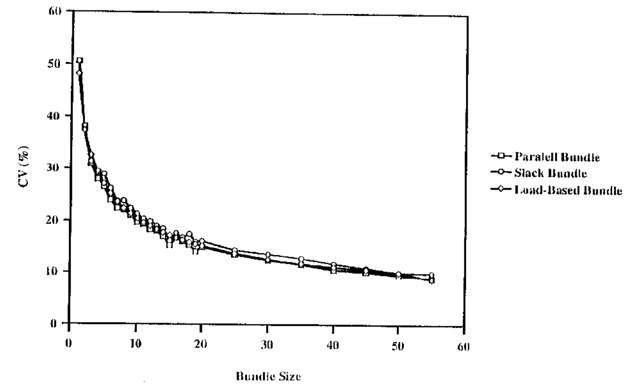

Figure 7. CV of bundle breaking strength ("B Cotton")

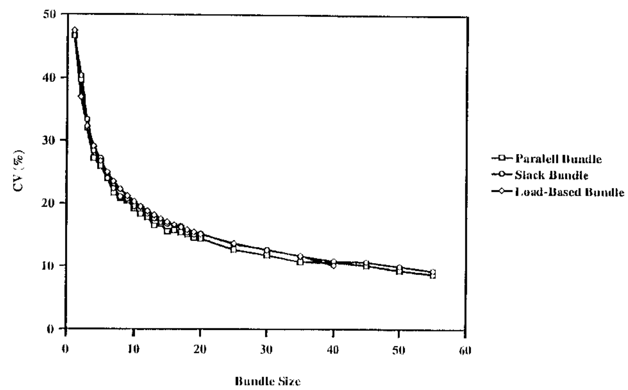

Figure 8. CV of bundle breaking strength ("I Cotton")

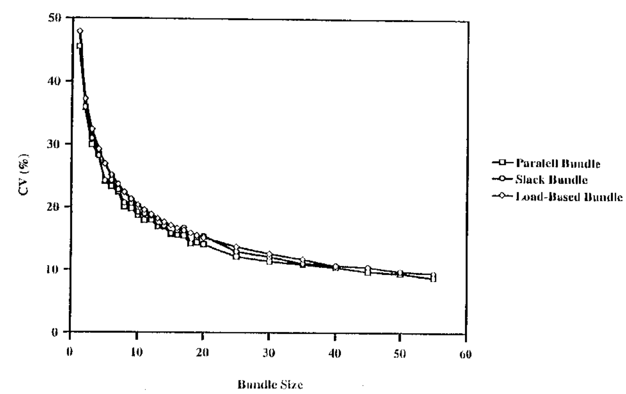

Figure 9. CV of bundle breaking strength ("T Cotton")