ESTIMATING THE ECONOMIC OPTIMAL RATE OF NITROGEN FERTILIZER: A BATTLE OF FUNCTIONAL FORM

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Abstract

Agricultural producers make fertilizer decisions based on recommendations from extension personnel and/or consultants established by the best available data; however, optimal nitrogen (N) recommendations can vary depending on the functional form used to estimate yield response functions. Applying too much or too little N fertilizer can negatively influence producer profits, as well as environmental conditions. The objective of this research is to evaluate different crop response models for cotton and compare the resulting economically optimal rates of N fertilizer. The overall conclusion, when analyzed by year, is that the appropriate functional form differs by year. For 2006, 2007, and 2008, the preferred functional forms were the square root, linear, and quadratic functional forms, respectively. In 2008, the difference in the expected revenue above N costs between the asymptotic maximum and the economic optimal was approximately $1 per acre. There was little difference between the asymptotic maximum and the economic optimal.

Introduction

Agricultural producers make fertilizer decisions based on recommendations from extension personnel and/or consultants established by the best available data; however, optimal nitrogen recommendations can vary depending on the functional form used to estimate yield response functions. Applying too much or too little nitrogen fertilizer can negatively influence producer profits and environmental effects.

Although there are numerous functional forms available, some are more widely used by researchers than others. Griffin et al. (1987) explored twenty functional forms based on a review of literature; however, few of these functional forms are used by agronomic and economic researchers to estimate crop response models. Linear, quadratic, and linear plateau models are common in agronomic research investigating crop yield response. Economists tend to explore other types of models, such as the Mitscherlich-Baule and the square root. The most appropriate functional form is dependent on the available data.

Depending on the crop and the type of research, different functional forms have been used extensively over the decades to estimate crop yield response. In the early 1850's, von Liebig introduced "the law of the minimum", which states that "the yield of any crop is governed by any change in the quantity of the scarcest factor called the minimum factor and as the minimum factor is increased the yield will increase in proportion to the supply of that factor until another factor becomes the minimum (Redman and Allen, 1954)." Anderson and Nelson (1975), along with Lanzer and Paris (1981), were pioneers in the use of linear response and plateau functions in agricultural economics. Up until the 1970's, most considered the response between yield and fertilizer to be smooth. Anderson and Nelson (1975) state that a new crop response model needs to "1) lead to reasonable accurate estimates of the optimal fertilizer rates for various decision rules; 2) produce a satisfactory goodness-of-fit to the data; 3) be easily adopted to obtain results based on the average of a number of experiments; and 4) be amenable to easy calculation (Anderson and Nelson, 1975)." They developed a series of models that were all similar to the linear-plateau model. Lanzer and Paris (1981) conclude that other functional forms may have advantages over the polynomial form of the response function that had been traditionally utilized by agricultural economists.

Since Anderson and Nelson (1975) and Lanzer and Paris (1981), agricultural economists have embraced a wide variety of functional forms for crop response functions, such as, but not limited to, the quadratic, square-root, linear von Liebig, Mitscherlich-Baule, and nonlinear von Liebig (Ackello-Ogutu et al., 1985; Berck et al., 2000; Berck and Helfand, 1990; Boyer et al., 2010; Frank and Beattie, 1990; Griffin et al., 1987; Grimm et al., 1987; Llewelyn and Featherstone, 1997; Paris, 1992a; Paris, 1992b; Tembo et al., 2008; Tumusiime et al., 2011). Berck and Helfand (1990) and Paris (1992b) estimated linear response plateau models using a switching regression model, based on the technique outlined in Maddala and Nelson (1974). Tembo et al. (2008) took it a step further and utilized the switching regression model to estimate wheat yield response functions in Oklahoma with a random year effect and a stochastic plateau.

Agronomic research tends to use similar functional forms regardless of crop type being studied. Cerrato and Blackmer (1990) compared and evaluated five models (linear plateau, quadratic plateau, quadratic, exponential, and square root) for estimating corn yield response. They found that the models predicted similar maximum yields but different economically optimal rates of nitrogen fertilizer. The quadratic plateau model was identified as the most appropriate model given the available data. Agronomic research tends to use either a linear or quadratic functional form for cotton yield response, depending on the type of treatment (Boquet et al., 2004; Reiter et al., 2008; Torbert and Reeves, 1994). Linear and quadratic plateau models are also used to calculate optimum fertilizer rates (Boquet et al., 2009; Bronson et al., 2001); however, authors rarely explain the factors that influence the decision to choose one functional form over another. The objective of this research is to evaluate different crop response models for cotton and compare the resulting economic optimal rates of nitrogen (N) fertilizer.

Materials and Methods

The data are from an experiment conducted at the Wiregrass Research and Extension Center in Headland, Alabama during crop years 2006, 2007, and 2008. It is a traditional cotton response trial with five N rates (0, 30, 60, 90, and 120 lb N ac^{-1}) applied at sidedress to cotton. The rotation used was continuous cotton with a rye cover crop. The cover crop did not receive nitrogen fertilizer. Table 1 contains the summary statistics for the entire sample, the sample by year, and the sample by N rate and year. The average rainfall between planting and harvesting was 13.4 inches, 13.4 inches, and 19.4 inches in 2006, 2007, and 2008, respectively. All plots were fertilized with phosphorus (P) and potassium (K) if needed.

Table 1. Summary statistics for entire sample, by year, and by nitrogen rate and year

Variable	Variable Description	All Years (n = 60)		2006 (n = 20)		2007 (n = 20)		2008 (n = 20)	
		Mean	Std Dev	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
lintlb	Cotton lint yield in lb ac^{-1}	1365	352.25	1443	260.87	1149	346.77	1504	348.24
Nitrogen (N) Rate: lb of N ac^{-1} of cotton		All Years (n = 12 per nrate)		2006 (n = 4 per nrate)		2007 (n = 4 per nrate)		2008 (n = 4 per nrate)	
		Mean	Std Dev	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
0		911	293.55	1139	344.97	676	178.19	918	149.54
30		1244	249.73	1399	111.89	943	159.08	1390	96.34
60		1450	267.88	1484	254.35	1168	8.15	1698	93.45
90		1564	227.1	1560	175.92	1352	112.49	1781	150.34
120		1657	91.92	1632	97.79	1607	48.14	1732	86.42

For nitrogen rates above zero lb N ac^{-1}, the average yields were above 1000 lb ac^{-1}, with the exception of the 30 lb N ac^{-1} rate in 2007. Based on county averages from USDA-NASS, the average cotton lint yield in Henry County, AL, where the experiment was located, in 2006, 2007, and 2008 was 459, 436, and 572 lb ac^{-1} respectively. The experimental yields are more than double the average county yields in the same years. This is most likely due to irrigation, as well as the intensive management of the experimental plots and differences in management techniques, such as the use of cover crops. Furthermore, the ginning percentage was 0.47, which is higher than the typical ginning percentage of approximately 0.40, due to use of a table-top micro-gin. Figure 1 displays the actual yield data as box plot for all years and by year. In 2006, there was high variability at the zero rate of nitrogen, which may have been due to previous treatments on these experimental plots. The variability in yields does decrease in 2007, and there is minimal variability in yields in 2007 at the 60 lb N ac^{-1} rate, as well as lower variability at the highest nitrogen application rate (120 lb N ac^{-1}). Variability between nitrogen application rates diminished in 2008.

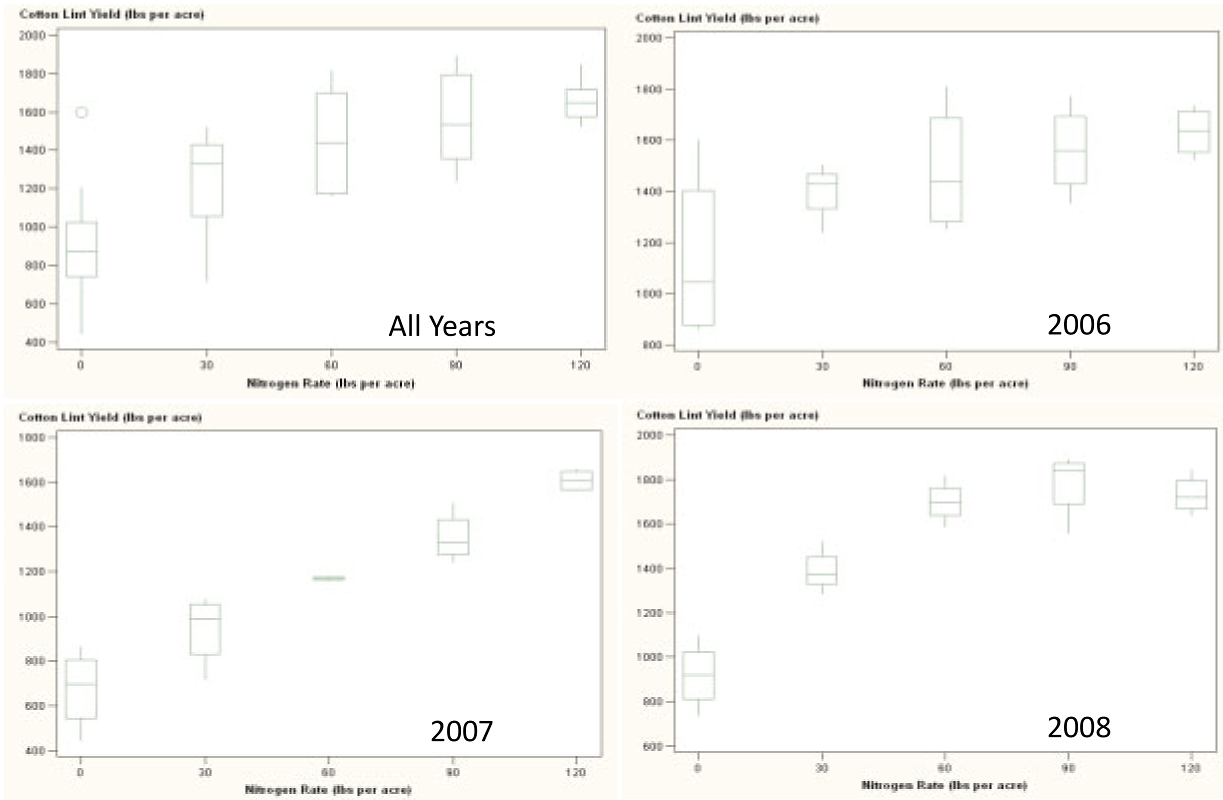

Figure 1: Box charts for cotton lint yields in lb ac^{-1} by nitrogen rate for all years and by year.

Following Tumusiime et al (2011), it is assumed that cotton producers are risk neutral and their objective is to maximize profits from cotton. The maximization equation is as follows: $\max_N E(\pi|N) = pE[Y_{ij}] - rN$. The expected profits from cotton (π) is equal to the price producers receive for cotton lint (\$ lb^{-1}) multiplied by the expected cotton lint yield minus the price of nitrogen (\$ lb^{-1}) multiplied by the pounds of N applied per acre. The maximization equation is subject to the expected yield as a function of the applied nitrogen, where nitrogen is greater than or equal to zero.

The following five cotton yield response models were fit to the data, as shown in Table 2: linear, quadratic, square root, Mitscherlich-Baule, and linear-plateau. These five models were chosen due to their prominent use in both agronomic and economic research. For all five models, the dependent variable is cotton lint yield in pounds per acre (Y_{ij}) for year i and plot j and the independent variable is the pounds of nitrogen fertilizer applied per acre (N_{ij}) for year i and plot j.

Table 2. Functional forms and corresponding yield response models

Functional Form	Equation
Linear	$Y_{ij} = \alpha_0 + \alpha_1 N_{ij} + \gamma_t + \varepsilon_{ij}$
Quadratic	$Y_{ij} = \alpha_0 + \alpha_1 N_{ij} + \alpha_2 N_{ij}^2 + \gamma_t + \varepsilon_{ij}$
Square Root	$Y_{ij} = \alpha_0 + \alpha_1 N_{ij}^{\frac{1}{2}} + \gamma_t + \varepsilon_{ij}$
Mitscherlich-Baule	$Y_{ij} = \alpha_0 * (1 - exp(-\alpha_1 * (\alpha_2 + N_{ij}))) + \gamma_t + \varepsilon_{ij}$
Linear-plateau	$Y_{ij} = \min(\alpha_0 + \alpha_1 N_{ij}, yp^*) + \gamma_t + \varepsilon_{ij}$

In all of the models, except the Mitscherlich-Baule, the parameter α_0 is the intercept (cotton lint yield if the nitrogen rate is zero) and, in the linear model, α_1 is the increase in cotton lint yield with a one pound increase in the nitrogen rate. In the quadratic model, $a_1 + 2a_2N$ (the first derivative) is the estimated slope, and can be interpreted as the first pound of nitrogen applied increases yield by a_1, while the second pound is worth less (if the signs on the parameters are consistent with the quadratic functional form). In the linear-plateau model, the plateau yield (yp^*) and the asymptotic optimal nitrogen rate (N*) are obtained when the model is fit to the data. If the marginal value product (MVP) is greater than the marginal factor cost (MFC), the N* is the level required to reach the plateau or zero. In all five models, the year random effect (γ_i) and the random error term (ε_{ij}) are considered to be normally distributed. When the models are estimated by year, the year random effects drop out of each of the models. The economically optimal nitrogen rate (N^e) is calculated by setting the first derivative equal to the fertilizer-to-cotton lint price ratio. The fertilizer-to-cotton lint price ratio of 0.754 is based on nitrogen prices of 0.66 \$ lb^{-1} and on cotton lint prices of 0.875 \$ lb^{-1}.

Results and Discussion

The estimates for each of the functional forms are found in Table 3. Due to space limitations, the parameter estimates are displayed for the functional forms with the smallest Akaike Information Criterion (AIC).

Table 3. Summary of regression results for cotton yield response functions across all years and by year.

Parameter	All Years	2006	2007	2008
	Square Root	Square Root	Linear	Quadratic
a_0	8.99**	11.43***	6.95***	9.19***
	(0.975)	(0.854)	(0.409)	(0.486)
a_1	0.693***	0.446***	0.076***	0.188***
	(0.058)	(0.110)	(0.076)	(0.019)
a_2				-0.001***
				(0.0002)
-2 Log-Likelihood	242.9	82.1	59	58
AIC	250.9	88.1	65	66

Note: Numbers in parentheses are standard errors.
Level of significance is denoted by *** for 1%, ** for 5%, and * for 10%.

The parameter estimates are all statistically significant at the 1% significance level, except for the slope parameter (a_1) for the square root model, which is significant at the 5% level. It is important to note that the lint yields were scaled by 100. For example, for the square root functional form in 2006, the intercept is 11.43 in 100 pounds of cotton lint yield per acre (1143 lb ac^{-1}).

For the complete data set and 2006 alone, the square root model assumes that yields increase over time, but at a declining rate. There is an economically optimal level of nitrogen in the square root model; however, it is far greater than the maximum amount of fertilizer applied in the experiment. For all years combined, assuming the economic and asymptotic optimal nitrogen rates are the maximum amounts applied in the experiment (120 lb N ac^{-1}), the estimated yield is 1658 lb ac^{-1} and the expected returns above N costs, excluding application, is 1372 \$ ac^{-1}. For 2006, assuming the economic and asymptotic optimal nitrogen rates are the maximum amounts applied in the experiment (120 lb N ac^{-1}), the estimated yield is 1631 lb ac^{-1} and the expected returns above N costs, excluding application, is 1348 \$ ac^{-1}.

In 2007, the linear model assumes that yields will continue to increase with each added pound of nitrogen. In both the square root and the linear model, profits will also continue to increase. Based on the model results, applying zero lb N ac^{-1} yields approximately 695 lb ac^{-1}, which is slightly higher than the mean actual yield in 2007 (676 lb ac^{-1}) for the application of zero nitrogen. Assuming the economic and asymptotic optimal nitrogen rates are the maximum amounts applied in the experiment (120 lb N ac^{-1}), the estimated yield is 1607 lb ac^{-1} and the expected profit is 1326.93 \$ ac^{-1}.

In 2008, the quadratic model assumes that yields will increase until a given maximum and then decrease with the application of additional nitrogen. The asymptotic maximum nitrogen rate is 93 lb N ac^{-1}, and the economic optimum is 89 lb N ac^{-1}. The estimated yield at the asymptotic maximum is 1802 lb ac^{-1}, and the expected profit is 1515 \$ ac^{-1}. At the economic optimal, the estimated maximum yield is 1800 lb ac^{-1}, and the expected profit is 1516 \$ ac^{-1}. The difference in the expected profit between the asymptotic maximum and the economic optimal is 1 \$ ac^{-1}, which is negligible. With a change in the fertilizer-to-cotton lint price ratio, the economic optimal rate changes very little, ranging from 91.68 lb N ac^{-1} at a price ratio of 0.30 to 88.22 lb N ac^{-1} at a price ratio of 1. Assuming a producer applies 90 lb N ac^{-1} instead of the economically optimal rate, the economic losses across all years and for 2006, 2007, and 2008 individually, would be 70 \$ ac^{-1}, 37 \$ ac^{-1}, 180 \$ ac^{-1}, and 7 \$ ac^{-1}, respectively.

Summary

Choosing the most appropriate functional form to estimate crop response models is important in agronomic and economic research. Producers make their nitrogen application decisions based on fertilizer recommendations. Over-applying nitrogen can affect the profits received by producers, as well as increase the potential for negative environmental impacts. Different functional forms can give very different economic and asymptotic optimums; however, the data dictates the functional form. The results depend on whether models are estimated for all years or by year. The square root functional form is the preferred model when all data is considered and in 2006. The linear functional form is preferred in 2007, and the quadratic in 2008.

This study raises additional questions. First, prior to the initiation of this experiment, the plots were in conventional tillage, without a cover crop. These data seem to show that it takes two years for the variability in yields to stabilize and for the cotton yield response to assume a quadratic functional form, which is the expected functional form. Additional research is needed to determine if this trend is common in other locations across Alabama and within other experiments, or is it exclusive to this experiment. Secondly, consideration should be given to whether or not additional levels of nitrogen are needed in similar experiments in order to better estimate the crop yield response. Third, does the use of growth regulators in cotton production increase the amount of nitrogen that can be applied to cotton without a yield decrease? Rank growth in cotton, due to excess nitrogen, may cause yield declines; however, due to the use of growth regulators, rank growth is contained, potentially causing cotton yields to increase with higher rates of nitrogen. These questions are important in determining economic optimal rates of nitrogen fertilizer and may have a place in future research. Although the topic of functional form has received considerable attention over the last 50 years, there is still room for improvements in choosing the proper functional form, particularly by crop and location, as well as the analysis of a wide range of functional forms that are underutilized in agronomic and economic research.

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