

WHERE DOES YOUR SHORT FIBER COME FROM? A SIMPLE DEMONSTRATION USING CUISENAIRE® RODS

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Abstract

Cotton short fiber is defined in the U.S. as fibers having length less than 0.5 inches. Because short fibers are so deleterious to our customers in the textile-manufacturing sector, the preservation of cotton quality through the control and elimination of short fiber has been the focus of enormous attention amongst breeders, growers, and ginners. Research on cotton short fiber in the U.S. is a team effort, involving substantial resources of the National Cotton Council of America, Cotton Incorporated, USDA, and State and private universities working closely with cotton industries across the Cotton Belt. In the future, short fiber content (SFC) of cotton will be an increasing challenge to the marketing and utilization of cotton. Pioneering research on cotton short fiber has shed light on its origin and its relation to fiber breakage. However, fiber breakage mechanisms are controversial, and raise new questions about the relative contribution of fiber properties and ginning technology to SFC. In this demonstration, well-established educational aids called Cuisenaire® Rods were used to visualize experimentally the effect of fiber breakage on the cotton fiber length distribution during post-harvest processing. This simple manipulative model showed that fiber breakage produced changes in fiber length distributions that were qualitatively similar to known changes that occur in seed cotton fiber as it is processed into the bale. It was also illustrated that the shape of the final distribution was related to the amount and type of breakage that was imposed.

Introduction

Physical concepts of dimension and mathematical conservation of quantity can be learned in early childhood by comparing the lengths of objects. If you have visited a kindergarten or nursery school class recently, you may have found children playing with a set of popular learning aids called Cuisenaire® Rods. These objects are hands-on educational toys of a type known as “manipulatives.” The Merriam-Webster Online Dictionary defines Cuisenaire Rods as, “any of a set of colored rods usually of 1 centimeter cross section and of 10 lengths from 1 to 10 centimeters that are used for teaching number concepts and the basic operations of arithmetic.” Each rod length is associated with a distinctive color for easy identification. If two or more rods are laid end-to-end, their total length can be readily determined to be less than, equal to, or more than the length of a long rod laid alongside. For instance, a 5-rod and a 4-rod together have the same length as a 9-rod. That length is also equal to the combined length of three 3-rods, etc. These educational toys are well-established tools for teaching mathematical concepts of quantity to pre-school and kindergarten-aged children. They are commonly used to teach addition, subtraction, multiplication, fractions, squares, area, measurement, conservation of quantity, and many other fundamental aspects of mathematics (Cuisenaire 1957).

Lest the reader’s intellect be offended by the elementary nature of the toys, the author hastens to point out that another important application of Cuisenaire rods involves the teaching of foreign languages to adults (Mullen 1996). In this case, the rods can represent phonemes or syllables, while the lengths and colors can denote inflection, stress, number of letters, or so forth. For instance, exercises based on the substitution of rods can be devised to correspond to conjugation or tense of verbs, to the use of prepositions, to contractions, or to other syntactical relationships between spelling, pronunciation, and meaning (Gattegno 1962). The successful utilization of Cuisenaire Rods in the teaching of English as a foreign language to highly intelligent adults has been reported in the international banking industry. This author has no expertise in languages, so the present work focused entirely upon the mathematical aspects of Cuisenaire Rods to illustrate basic quantitative relationships between properties of fiber length.

A crucial research issue for cotton fiber length is the origin of short fiber. One important hypothesis is that the amounts of short fiber that are significant in textile processing originate essentially from fiber breakage associated with the mechanical handling necessary to gin and clean cotton as a part of bale production, to prepare bale fiber for spinning, or to process a fiber sample as part of a laboratory measurement of length (Robert 2005b). In order for this hypothesis to become established scientifically, two things are necessary: (1) an absence of disproof in the face of aggressive skepticism, and (2) a demonstration of feasibility. Cotton physiologists and fiber scientists who

studied individual-fiber lengths on cottonseed repeatedly have reported that short fiber was not prevalent in normal growth because lint fibers tended to attain the same full length, more or less, on a seed (Robert 2005a). Therefore, the issue was the feasibility of the hypothesis. The present work attempted to provide a simple qualitative and quantitative illustration of conceptual feasibility for the breakage hypothesis.

Materials and Methods

The standard set of Cuisenaire objects consists of ten rods, ranging from one to ten units (cm) in length. The standard colors are shown in Figure 1. Similar to fibers, Cuisenaire rods have an exaggerated aspect ratio. This means that one of the three dimensions (length) is greater than the other two (cross-sectional) dimensions. In the present work, Cuisenaire Rods were considered as models of fiber length groups, with each unit of rod length corresponding to 1/8-inch of cotton fiber length at the top of the group. Collections of rods were used to model samples of cotton fiber. Arrays of the rods sorted by length were used to model cotton fiber length histograms, such as would be produced by the Suter-Webb method. Rods of length four or less represented short fiber.

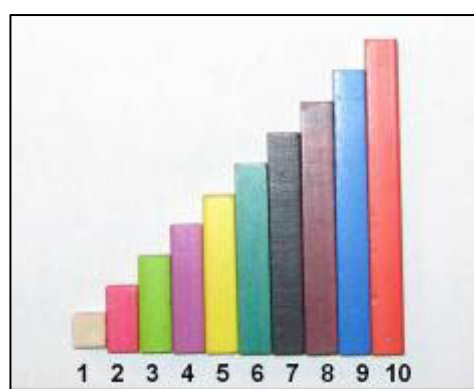


Figure 1. Standard Cuisenaire Rods.

If there is not a substantial amount of short fiber in normal seed cotton, then what is the origin of all that short fiber in the bale? Is it possible that it could be due to excess fiber breakage (not counting the one ginning break per fiber necessary to remove the fiber from the seed)?

In this work, the physics of fiber breakage was investigated by creating a simple, highly uniform distribution of a small number of rods to coarsely model a seed cotton fiber length distribution, and then “breaking” some of the rods (fibers) by replacing each of them with two rods that together had the same total length as the original rod. The broken “fibers” were then rearranged by length in the master array. In this manner, the effect of partial breakage on the overall length distribution could be readily visualized. The author’s three granddaughters, aged three to six, were enlisted to demonstrate the rods and perform some of the manipulations. In Figure 2, Nina has discovered that a 4-rod plus a 3-rod together had the same length as a 5-rod plus a 2-rod. We could also infer from this that a 7-rod (or fiber) could be cut (or broken) into either of these combinations. This works because length is conserved in breakage (Robert 2005b). For the purposes of this work, rods of two additional non-standard lengths (eleven and twelve units) were fabricated (cf. Stern 1949), as shown in Figure 3. Firstly, a simple fiber distribution was constructed. This distribution was rather uniform and had almost no short fiber. It was not unlike the reported shape of seed fiber distributions (Robert 2005a).



Figure 2. Nina has discovered that $5 + 2$ is equivalent to $4 + 3$.

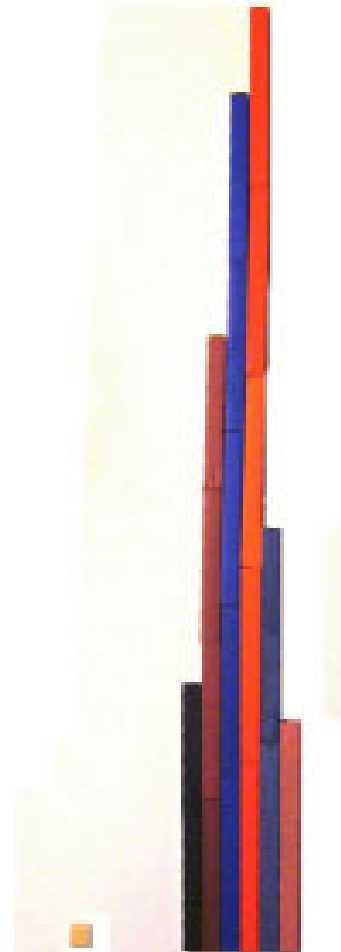
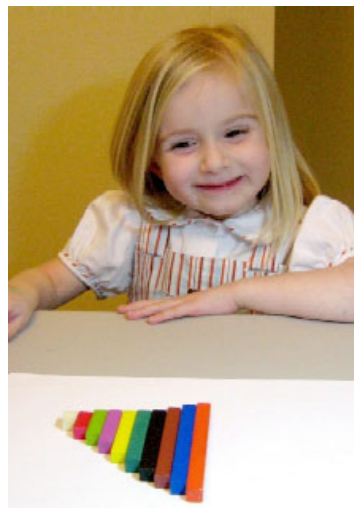


Figure 3. Lizzie, age four, learned how to make length arrays. Molly, age six, helped construct an original starting array that was used as the reference distribution throughout this study.

Next, the children investigated the effect of breaking one fiber out of each length group (i.e., Cuisenaire Rod length) into two fragments by “random” breakage. The “unbroken” and “to-be-broken” components of the original distribution are shown in Figure 4. The right side of Figure 5 shows only the fibers to be removed from the original distribution by breakage. For each length, the position of the break was varied according to the throw of a die

(Figure 6), which determined the length of the shorter of the two broken fragments. An additional rule was used to forbid breakage into the “1” unit length. That rule reflected the frictional “gripping” effect of cotton, which requires a grip length of more than 1 cm to develop enough force to break the fiber rather than allow it to slip out of the grip. For each of the fiber lengths to be removed from the original distribution by breakage, the original fiber was replaced with two fibers having same total length. This could be accomplished conveniently by laying the two “broken” fragments upon the rods in the original array to verify that the total length was the same (Figure 7).

Then the children were asked to rearrange all the broken fragments and the unbroken rods back into one master array. The result was the daughter array shown in Figure 8. The shape of this daughter array (i.e., partially-broken array) differed from that of the parent array due to the breakage that had been imposed. Note the introduction of short fiber and the bimodal shape.

This methodology was replicated upon the same initial array to get some idea of the variability involved in the final distribution. Other types of breakage rules were also explored. These included: the maximum short fiber content theoretically possible from this breakage scenario; breakage to produce the minimum short fiber possible; the breakage of all the fibers instead of only one from each length group; breakage as near as possible to the middle of each fiber instead of at random positions along the extent. and breakage of differing number of fibers from the original array governed by probability (17% or 50%).

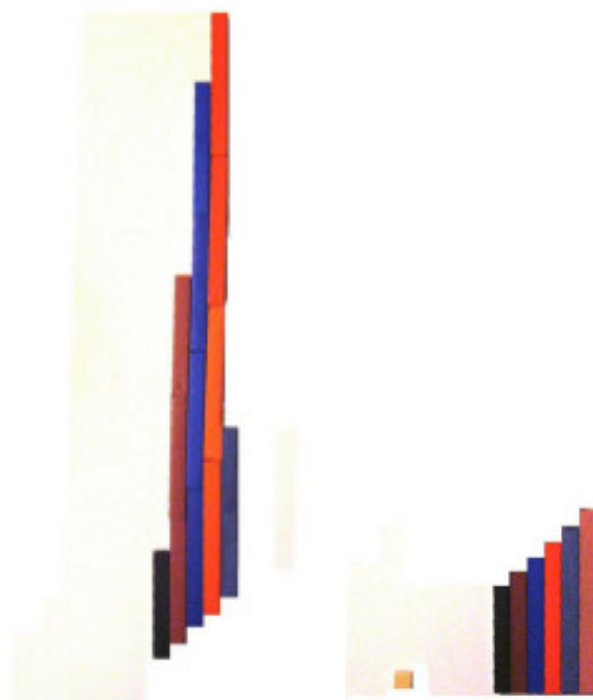


Figure 5. The starting distribution is divided into the “not-to-be-broken” (left) and “to-be-broken” fractions. One fiber from each populated length group is shown on the right side. Each rod in that group will be “broken” by replacing it with two rods having the same total length.

Results

Results of the properties of the arrays are shown in Table I. The same original unbroken distribution was used in all of the scenarios that were investigated. The properties of that distribution are identified as scenario A. Scenario B consisted of random breakage along the extent of the fibers, governed by throws of the die. The first throw determined whether the fiber would be broken (value of 1) or remain unbroken (value of 2 to 6). If the fiber were to be broken, the next throw would determine the length of the short fragment. If the number thrown was larger than the range of possible values (*e.g.*, throwing a 6 for a 6-rod, instead of a 2 or 3), the die was thrown again until the length was decided. Scenarios C, D, E, and F used one rod from each length group (except the unit length) for replacement. In scenario C, the breakage pattern leading to the minimum SFC was deduced. Likewise, in scenario F, the pattern having maximum SFC was deduced. In scenario E, on the other hand, each fiber was “broken” as close to its midpoint as possible. No throw of the die was involved in these three scenarios (C, E, and F) because they each represented a unique daughter distribution. In scenario D also, the number of to-be-broken fibers (one from each length group) had already been chosen, so no initial throw of the die was necessary. The subsequent throw(s) to determine the fragment lengths, however, were required. Scenario G was similar to B, except that the breakage decision was decided positively for the range 1 to 3 on an initial throw of the die. In scenario H, every fiber (except the 1-rod) was broken. Only the lengths of the fragments were determined by chance.

Table I. Properties of the Unbroken and Broken Arrays

Parameter	Mean Length by mass	Broken Fiber Content by mass	Short Fiber Content by mass	Mean Length by number	Broken Fiber Content by number	Short Fiber Content by number
Symbol	ML_{mass}	BFC_{mass}	SFC_m	ML_{num}	BFC_{num}	SFC_n
Units	(in)	(%)	(%)	(in)	(%)	(%)
Breakage Scenario						
A: Original Unbroken Distribution	1.107	0	0.6	1.038	0	5.0
B: Random Break at 17% prob. *	1.027	16.3	6.2	0.892	26.5	18.4
C: Break 1 of each Length, Min SFC	0.928	32.4	4.0	0.784	46.2	15.4
D: Break 1 each Length at Random *	0.940	32.4	10.6	0.784	46.2	26.7
E: Break 1 each Length, in Middle	0.909	32.4	11.4	0.784	46.2	23.1
F: Break 1 each Length, Max SFC	0.919	32.4	18.2	0.784	46.2	34.6
G: Random Breakage at 50% prob. *	0.840	53.0	18.3	0.673	66.3	37.1
H: Random Breakage at 100% prob. *	0.621	99.4	33.8	.502	97.4	53.6

* Average of replication by 10 random trials

Table II. Standard Deviation of Properties of Random Arrays

Parameter	Mean Length by mass	Broken Fiber Content by mass	Short Fiber Content by mass	Mean Length by number	Broken Fiber Content by number	Short Fiber Content by number
Symbol	ML_{mass}	BFC_{mass}	SFC_m	ML_{num}	BFC_{num}	SFC_n
Units	(in)	(%)	(%)	(in)	(%)	(%)
Breakage Scenario						
B: Random Break at 17% prob.	0.041	8.1	3.3	0.063	11.4	6.4
D: Break 1 each Length at Random	0.012	*	2.21	*	*	3.1
G: Random Breakage at 50% prob.	0.064	12.1	5.9	0.058	10.5	7.25
H: Random Breakage at 100% prob.	0.019	*	3.2	*	*	3.5

* Not applicable in this breakage scenario.

Discussion

Breakage over the range from zero to 100% probability manifested a decrease in mean length of about a factor of two. Scenarios C, D, E, and F produced very different results, even though in each case, the exact same fibers were broken, and, therefore, the BFC_{mass} , BFC_{num} , and ML_{num} were the same. Nevertheless, the SFC_{mass} ranged from 4.0% to 18.2%, while the ML_{mass} varied from a maximum of 0.940 inch to as little as 0.909 inch (Table I). These differences are dependent strictly upon the manner in which the fibers were broken, not upon which fibers were broken. In other words, only the replacement of the Cuisenaire rods determined variations in the outcome, because the same rods were replaced in each case. The standard deviation of SFC in the demonstration due to random breakage was about a couple of percent (Table II). That is roughly similar to levels observed in practice. It is especially interesting to note that ratio of BFC to SFC ranged from a minimum of about two in scenario F to a maximum of about eight in scenario C. This compared roughly with the wide range of about three to seven that has been noted in experimental work. Given the crudeness of this model, it should perhaps have been more surprising that the known experimental norms exhibited such a wide independence between breakage and short fiber. In the model, this could have been due to the relatively small number of fibers ($N = 20$) in the original array, and to the crude classification scheme (12 intervals). However, in sophisticated fiber length measurement systems, a smaller length interval and a larger specimen size of 5,000 to 20,000 fibers are commonplace. So, the wide variation in experimentally observed SFC could be a fundamental feature of the nature of fiber breakage that was also captured by this simple demonstration.

Length arrays of Cuisenaire Rods inherently represent mass-biased histograms of length, rather than number-biased distributions. They are not a perfect model due to the rather large interval size (1/8-inch), and the relatively small number of elements that can be reasonably manipulated in a hands-on demonstration. In addition, the difference between the lengths at the top (rather than at the midpoint) of each interval representing the mass is an obvious obstacle to precision and accuracy. The utility of the model, however, lies in its conceptual simplicity. In the present analysis, the difference between the interval tops and midpoints was ignored for all parameters except the mean lengths, and the rod length was used to describe both the length and mass. In calculating the mean lengths, however, the midpoints were used. The uncertainties associated with this artifact are small compared to the ranges of values being demonstrated.

Is SFC a property of cotton? Or is it an artifact of processing? Breeders are quick to point out that the same cotton will produce a different SFC with different gin, cleaning, or measurement treatments. So they will say the gin tears up the cotton. But the ginner is equally certain that the same ginning, processing, or measurement treatment can produce different SFC's for different cultivars. So they want to believe that it is the cotton that goes to pieces in the gin. The answer is probably that both are right. Fiber breakage is an interaction term between fiber properties and processing treatment. The present work demonstrated that there is a huge potential for variability in SFC, even when similar (identical) collections of fibers are broken. The statistically favored position of breakage along the length of the fibers is the key to the variability and quality of length. In this oversimplified model, breakage of fibers in the middle seemed to be the worst case, both in terms of loss of mean length, as well as in the buildup of short fiber.

Conclusions

A visual-tactile demonstration of the effects of fiber breakage on fiber length distribution was developed around the use of Cuisenaire® Rods. The demonstration was simple enough for children to perform, but was able to capture the essential nature of fiber length and mass as conserved physical quantities. So where does your short fiber come from? This demonstration illustrates the way in which measured cotton short fiber originates from the **breakage of longer fibers** in ginning, cleaning, and length measurement.

Notes

- The Southern Regional Research Center is one of the facilities of the Mid South Area, Agricultural Research Service, U.S. Department of Agriculture.
- The name Cuisenaire® and the color sequence of the Rods are registered trademarks of ETA/Cuisenaire®, A. Daigger & Company, Vernon Hills, IL.
- Names of companies or commercial products are given solely for the purpose of providing specific information; their mention does not imply recommendation nor endorsement by the U.S. Department of Agriculture over others not so mentioned.

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Figure 6. The throw of a die was used to determine the size of the two fragments for each fiber (rod) that was “broken.” (replaced).



Figure 7. Each fiber in the “to-be-broken” category was replaced with two broken “fragments” having the same total length. Compare with the right side of Figure 5.



Figure 8. A final, partially broken array was formed by re-assembling the “broken” and “unbroken” rods into one array. This array had the same total length (mass) as the initial array, but a different shape (compare with Figure 4) and different length parameters. The uniform starting distribution became skewed to the low side as a fragment hump developed. The fibers lost some mean length as the result of the breakage, while substantial short fiber (red, green, and purple) was generated. The degree of intensity of breakage was approximately 1/3 by mass.