STOCHASTIC DIFFERENTIAL EQUATION MODEL FOR COTTON FIBER BREAKAGE

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Abstract

A stochastic differential equation (SDE) model is developed for fibers undergoing breakage during textile processing steps. The SDE model generalizes a classic deterministic model for fiber breakage. Furthermore, SDE model compares well with Monte Carlo computations for different fiber breakage phenomena. Also calculations with the SDE model exhibit a bimodal distribution in fiber lengths which is commonly seen in data.

Introduction

In the cotton system, fiber breakage occurs in all mechanical processes undergone by the lint from the field to the spinning mill. The direct impact of breakage on fiber length distribution and on the incidence of short fibers represents a long-lasting concern in the cotton industry. The rich body of work dealing with modeling cotton fiber breakage (Byatt and Elting, 1958; Byatt, 1961; Pittman and Tallant, 1969; Tallant *et al.,* 1966; Robert and Blanchard, 1997; Robert *et al.,* 2000) attests to the importance of breakage phenomena in the cotton system. Building on this body of knowledge, our research introduces a new approach to modeling fiber breakage based on stochastic differential equations (SDE). The SDE model generalizes the classic deterministic model and offers a better understanding of fiber breakage and of the origination of different fiber length distributions (Simsek, 2007).

Model Development

In the stochastic model, the fibers are grouped by length so that the length distribution can be considered as a population distribution. The SDE model is derived by considering the population process and breakage possibilities over a short time interval using stochastic modeling techniques described in (Allen, 1999; 2003; 2007).

In developing an SDE model, m populations of fibers are considered as functions of time t. The changes in the fiber populations are tabulated for a small time interval dt. Both the mean change and the covariance in the change for the small time interval are calculated. For example, consider the special case where m=8. Consider a fiber in the 7th group breaking into two fibers, one in group 5 and one in group 2. The change produced is:

$$\left(\Delta\vec{N}\right)^{7,5} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

and the corresponding term produced in the covariance matrix is:

$$C^{7,5} = \left(\Delta\vec{N}\right)^{7,5}\left(\left(\Delta\vec{N}\right)^{7,5}\right)^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The value of the expected change for the small time interval is calculated by summing the products of the changes with the respective probabilities. Based on the population change, the SDE model expresses the fiber length distribution as a function of time t. More details on the model's theory can be found in (Allen, 1999; 2003; 2007).

Experimental Evaluation

Two cottons differing in maturity and strength were used to test the SDE model. Length distribution was obtained before and after carding and aggressive opening. The length distributions observed before each process were used as input for the SDE model and the model's outputs were compared to the distributions observed after processing. Figures 1and 2 depict the input length distribution and both observed and model-generated distributions of each of the two cottons (mature cotton and immature cotton, respectively).

The SDE-model outputs were obtained through successive approximation of the parameters. At this point of the research no optimization routine was used to obtain parameter estimates that correspond to the closest fit. All observed distributions used here were numerical and were obtained using the Uster® AFIS®.

It can be observed from the data in Figures 1 and 2 that the two cottons show distinct initial patterns. The immature-weak cotton (Figure 2) has an initial length distribution shape that is similar to that of the mature cotton after carding and extensive opening. It has been shown by Krifa (2006) that such shape features typically indicate advanced fiber breakage stages.

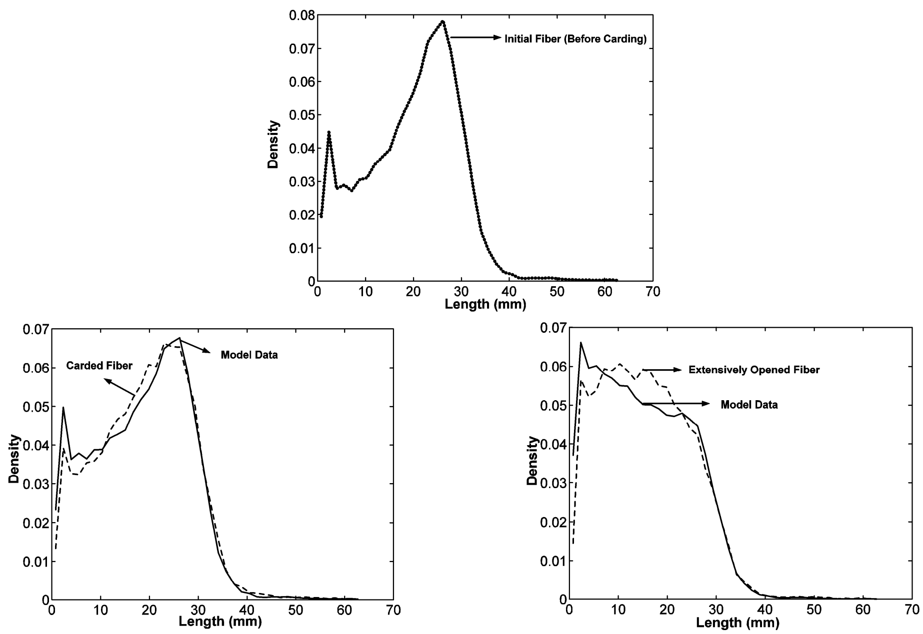

Figure 1: observed and model-generated length distributions for the mature-strong cotton.

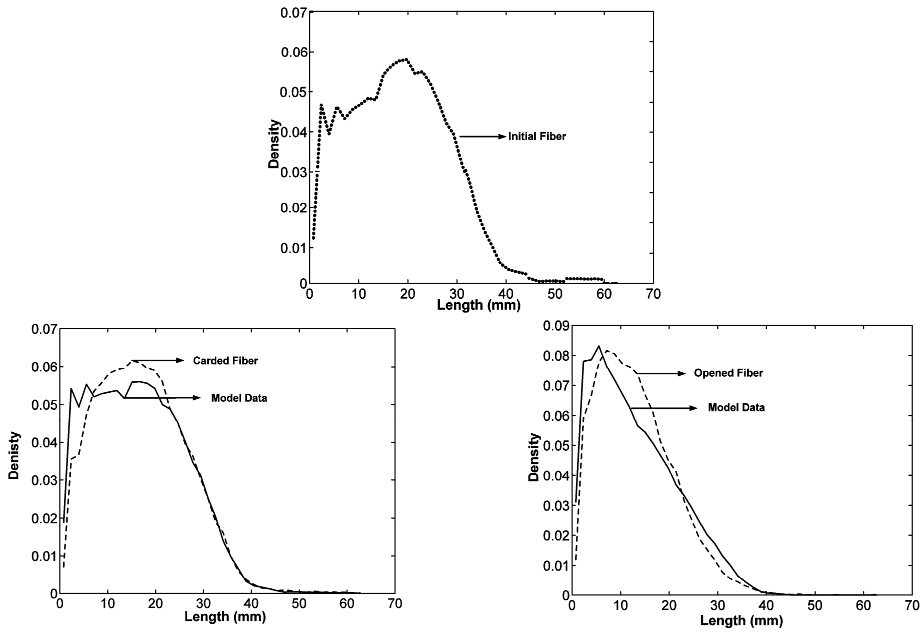

Figure 1: observed and model-generated length distributions for the immature-weak cotton.

When considering the distributions after carding and aggressive opening, both Figures show that the SDE model yields results that are rather similar to the real fiber distributions and that closely match the distribution shape features of both immature-weak and mature-strong cottons (distribution modality, Krifa, 2006). However, some differences in the outputs appear to be inevitable with the current model, even when adjusting the parameters.

It is for instance seen that the model distributions seem to be more broken than the real fibers. One possible explanation is that during the actual fiber processing very short fiber fragments may be eliminated from the material. This removal of the short fiber fragments is not taken into account in the SDE model. Work continues to take this issue into account and to include optimization routines that will allow better parameter estimates.

Conclusion

A stochastic differential equation model was developed for fibers undergoing breakage. The SDE model generalizes a classic deterministic model for fiber breakage and appears to adequately characterize various fiber breakage phenomena. Calculations with the SDE model generate numeric length distributions that closely resemble experimental results and that exhibit modality features commonly seen in data.

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References

Allen, E.J., Stochastic Differential Equations and Persistence Time of Two Interacting Populations. Dynamics of Continuous, Discrete, and Impulsive Systems, 5: 271-281 (1999).

Allen, E.J., "Modeling with Ito Stochastic Differential Equations", Springer, Dordrecht, The Netherlands. 2007.

Allen, L.J.S., "An Introduction to Stochastic Processes with Application to Biology", Pearson Education Inc., Upper Saddle River River, New Jersey. 2003.

Byatt, W.J. and Elting, J.P., Changes in the Weight Distribution of Fiber Lengths of Cotton as a Result of Random Fiber Breakage. Textile Res. J., 28 (5): 417-421 (1958).

Byatt, W.J., On Changes in the Weight Distribution Function of Artificial Fibers Due to Random Fiber Breakage. Textile Res. J., 31 (2): 171-174 (1961).

Krifa, M., Fiber Length Distribution in Cotton Processing: Dominant Features and Interaction Effects. Textile Res. J., 76 (5): 426-435 (2006).

Pittman, R.A. and Tallant, J.D., Random-Fiber Breakage Models. Textile Res. J., 39 (8): 787-789 (1969).

Robert, K.Q. and Blanchard, L.J., Cotton Cleanability. Part I: Modeling Fiber Breakage. Textile Res. J., 67 (6): 417-427 (1997).

Robert, K.Q., Price, J.B., and Cui, X., Cotton Cleanability – Part II: Effect of Simple Random Breakage on Fiber Length Distribution. Textile Res. J., 70 (2): 108-115 (2000).

Simsek, H., Stochastic Differential Equation Model for Cotton Fiber Breakage, M.S. thesis Mathematics and Statistics, Texas Tech University: Lubbock, TX, 47 p. (2007).

Tallant, J.D., Pittman, R.A., and Schultz, E.F.J., The Changes in Fiber-Number Length Distribution Under Various Breakage Models. Textile Res. J., 36 (8): 729-737 (1966).