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PREDICTION OF CRITICAL MIXING RATIO FOR COTTON POLYESTER BLENDED YARN STRENGTH

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Abstract

Yarn ultimate mechanical properties are important for prediction of fabric mechanical behavior and estimation of yarn complex quality. The prediction of single component cotton yarn strength based on the knowledge of fibrous bundles strength (HVI) and correction factors for fibers orientation and yarn packing density is now at disposal. In the case of two component yarns from cotton /polyester fibers mixtures is common that at some critical mixing ratio the minimal yarn strength appears. The prediction of critical mixing ratio is usually estimated by using of Hamburger model. Main assumptions are linearity of stress strain curves for both components and linear mixing rule. The main aim of this contribution is description of procedure for computation of critical mixing ratio based on the two methods:

1.The experimental values of strengths for set of yarns with various mixing ratios are at disposal. In this case the improved statistical method for estimation of critical mixing ratio is proposed. 2.Experimental information about strengths of both components, yarn fineness and yarn twist are at disposal. In this case the predicted cotton and polyester yarn strengths and break elongations are firstly computed and then the Hamburger model is used. The predicted strengths are computed from special model based on the modification of fiber strength by coefficients reflecting fibers orientation and yarn packing density.

Both methods are compared on the sets of experimental data according to their prediction ability.

Introduction

The mechanical properties of the ultimate yarn are important for the prediction of fabric mechanical behavior and estimation of yarn complex quality. In the case of two component yarns from cotton /synthetic fibers mixtures, at some critical mixing ratio the minimal yarn strength appears (Backer (1968)). The prediction of the critical mixing ratio is usually estimated by using of Hamburger linear mixing rule, is described for example in the work (Neckář (1998)). The classical estimator of critical mixing ratio is computed from experimental and predicted ultimate yarn characteristics composed from 100 % of individual components only. In the case when the strengths of the set of blended yarns are at disposal it is better to use linear mixing rule as regression model. The prediction of a single component cotton yarn strength is based on the knowledge of fibrous bundles strength (HVI) and correction factors for fiber orientation and yarn packing density and is described in the work (Křemenáková (2005)). For the prediction of the cotton yarn break deformation and synthetic yarn ultimate characteristics, the Uster Statistics (level 95%) can be used.

Theoretical part

The main assumptions of linear mixing model are the linearity of the stress strain curves for both components and linear mixing rule. The yarn strength of 100% for the first and 100% for the second components are marked σ_1 and σ_2, corresponding to the break elongations ε_1 and ε_2. The first yarn component has a lower fiber break elongation ε_1 [%] and $S_2(\varepsilon_1)$ [N.tex^{-1}] is the loading in the second component at deformation ε_1. From the linearity of the stress strain curves it is simply to derive, that

$$S_2(\varepsilon_1) = \sigma_2 \varepsilon_1 / \varepsilon_2 \quad (1)$$

The linear mixing means that the dependence of blended yarn strength is a piecewise linear function of mixing ratio as is shown on the fig.1. The relative portion of first component is g_1 and the portion of the second component is $g_2 = 1 - g_1$. Critical mixing ratio G_1 (Neckář (1998)) is defined as value of first component portion corresponding to the lowest yarn strength.

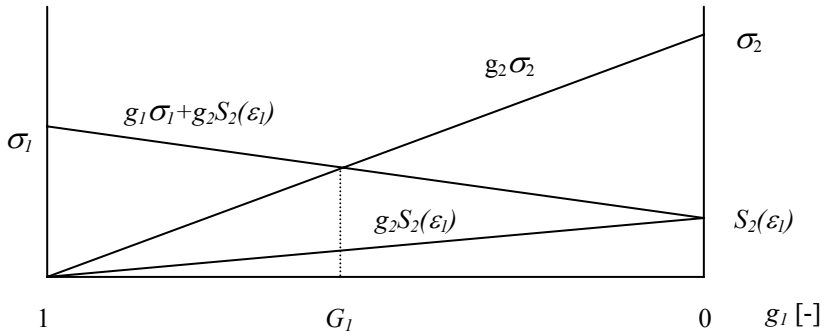


Figure 1 Relationship between blending ratio and yarn strength according Hamburger model

From fig. 1, it is straightforward that at critical mixing ratio is

$$G_1\sigma_1 + (1-G_1)S_2(\varepsilon_1) = (1-G_1)\sigma_2 \tag{2}$$

The critical mixing ratio is then expressed by the relationship

$$G_1 = \frac{\sigma_2(1-\varepsilon_1/\varepsilon_2)}{\sigma_1 + \sigma_2(1-\varepsilon_1/\varepsilon_2)} \tag{3}$$

In practice then, it is necessary to know three parameters $p_1 = \sigma_1$, $p_2 = \sigma_2$ and $p_3 = \varepsilon_1/\varepsilon_2$ only. The classical estimator of G_1 is computed from experimental ultimate characteristics of yarns composed from 100 % of individual components only. In the case when the strengths σ_y for the set of blended yarns is at disposal, it is better to use the linear mixing rule as regression model.

The regression model where $y = \sigma_y$ and $x = g_1$ has the form

$$y = (1-x)p_2 \qquad \text{for } x<G_1 \tag{4}$$

$$y = x(p_1 - p_2 p_3) + p_2 p_3 \qquad \text{for } x>G_1 \tag{5}$$

Parameters p_1, p_2 and p_3 can then be estimated by the least squares method. This approach leads to the so-called optimal estimator of G_1. The third possibility is to use eqn. (3) for the predicted yarn strengths of individual components. The estimator of G_1 computed by this approach is denoted as predicted.

Relative yarn strength of single component σ_y is generally expressed by the relationship

$$\sigma_y = \sigma_f \phi_{fy} = \sigma_b \phi_{by} = \sigma_f \phi_{fb} \phi_{by} \tag{6}$$

where σ_f is the relative fiber strength, σ_b is relative bundle strength, ϕ_{fy} is utilization of fiber strength in yarn, ϕ_{by} is utilization of fiber bundle strength in yarn and ϕ_{fb} is utilization of fiber strength in the fiber bundle. Various models for yarn strength prediction are described in works (Neckář (1998)), (Pan (1995), (1995a), (1993)) and (Zurek (1971)). Pan (1995) described fiber distribution as Weibull two-parameter type. Simple approximate relationships for utilization of fiber strength in fiber bundle was derived in (Křemenáková (2005)).

$$\phi_{fb} = u^u \exp(-u)/\Gamma(1+u)\,;\; u=0{,}909\, v_{\sigma_v} \tag{7}$$

In eqn. (7) is *Γ()* the gamma function and $v_{\overline{\sigma_v}}$ is variation coefficient of fiber strength. For utilization of fiber bundle in a yarn is valid (Pan (1995))

$$\phi_{by} = V_f \eta_\beta \tag{8}$$

where V_f is fiber volume fraction and η_β is the orientation factor. Fiber volume fraction V_f is computed from relationship

$$V_f[-] = 0{,}7(1 - 0{,}78\exp(-0{,}195T_y)) \tag{9}$$

where T_y is twist coefficient described by relationship

$$T_y\left[cm^{-1}tex^{1/2}\right] = 10^{-2}\sqrt{T[tex]}Z\left[\mathrm{m}^{-1}\right] = \alpha\left[m^{-1}ktex^{1/2}\right]\sqrt{1000}/100 \tag{10}$$

In eqn. (10) T is yarn fineness, Z is yarn twist and α is twist coefficient. Orientation factor η_β is computed according Pan (1995)

$$\eta_\beta = \frac{2\beta_D(1-\eta)+(1+\eta)\sin 2\beta_D}{4\beta_D} \tag{11}$$

Orientation factor is function of helix angle β_D and yarn Poisson ratio η. Helix angle β_D is defined according to equation (Pan (1995a))

$$\beta_D = arctg\left(10^{-1}T_y\sqrt{\frac{4\pi}{\rho V_f}}\right) = arctg\left(\alpha\sqrt{\frac{4\pi}{\rho V_f}}/\sqrt{10^3}\right) \tag{12}$$

where α is twist coefficient in [$m^{-1}ktex^{1/2}$], ρ is fiber density in [kgm^{-3}]. Poisson ratio η has the form (Pan (1995))

$$\eta = \frac{\sin^5\beta_D}{2\left(1-\cos^3\beta_D\right)\left(\frac{1}{2}\beta_D - \frac{1}{4}\sin 2\beta_D\right)} \tag{13}$$

For prediction of pure cotton yarn strength is better to use simpler the relationship (Křemenáková (2005))

$$\sigma_1 = \sigma_{HVI}\phi^*_{by} = \sigma_{HVI}\mu\eta^*_\beta \tag{14}$$

where σ_{HVI} is cotton fiber bundle strength and ϕ^*_{by} is the corrected utilization of fiber bundle strength in yarn, μ is the mean yarn packing density and η^*_β is corrected orientation factor. Packing density expresses in fact the fiber compactness in yarn. Packing density is defined as the ratio of fibers volume to the total volume of yarn. Prediction of yarn packing density is computed from the following model (Neckář (1998))

$$\frac{\left(\frac{\mu}{\mu_m}\right)^{5/2}}{\left[1-\left(\frac{\mu}{\mu_m}\right)^3\right]^3} = \frac{M\sqrt{\pi}}{2\mu_m^{5/2}\sqrt{\rho}}\left(ZT^{1/4}\right)^2 \qquad (15)$$

where M is parameter dependent on material and production technology. Corrected orientation factor η_β^* is computed by using of corrected angle β_D^*

$$\beta_D^* = arctg\left(\alpha\sqrt{\frac{4\pi}{\rho\,\mu}}/\sqrt{10^3}\right) \qquad (16)$$

Figure 2 illustrates some examples for the prediction of fiber bundle strength utilization in cotton yarn based on eqn. (14). In this figure, the influence on yarn fineness connected with yarn optimal twist and production technology on the utilization of fiber bundle strength in yarn is shown.

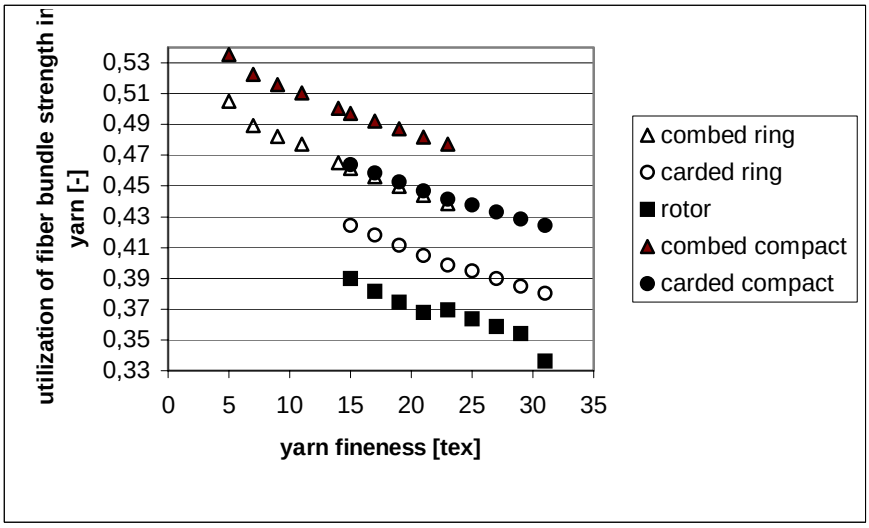


Figure 2 Utilization of fiber bundle strength in cotton yarn according eqn. (14)

For prediction of cotton yarn break deformation ε_1 and polyester yarns ultimate characteristics σ_2, ε_2 the following regression equations from the Uster Statistics (level 95%) were computed.

$$\varepsilon_1[\%]=5{,}7431\ (590/\ T\ [\text{tex}])^{(-0{,}0542)} \text{ for combed cotton yarn with fineness} \le 15 \text{ tex} \qquad (17)$$

$$\sigma_2\,[\text{cN/tex}]= 13{,}9816\ (590/\ T\ [\text{tex}])^{0{,}1733} \qquad (18)$$

$$\varepsilon_2\,[\%]=42{,}1952\ (590/\ T\ [\text{tex}])^{(-0{,}3930)} \qquad (19)$$

Experimental

The numerical values of critical mixing ratio for sets of blended yarns cotton / polyester and cotton / polypropylene (POP) are given in table 1.

Table 1 Critical mixing ratio for Cotton/PES Cotton/POP yarns

Raw material	Fineness [tex]	Classical critical mixing ratio [-]	Optimal critical mixing ratio [-]	Predicted critical mixing ratio [-]
PET/co	8	0,575	0,437	0,394
PET/co	12	0,527	0,474	0,448
PET/co	24	0,584	0,548	0,500
POP/co	20	0,507	0,428	0,472
POP/co	29,5	0,539	0,409	0,491
POP/co	45	0,549	0,434	0,518

The classical critical mixing ratio (here for cotton) was computed from experimental data by using linear mixing (3). The optimal estimator of critical mixing ratio was computed by eqn. (4) and (5). Predicted critical mixing ratio is based on the cotton yarn strength prediction from eqn. (14), cotton yarn deformation at break from eqn. (17), polyester strength and deformation at break from eqn. (18), (19) and polypropylene yarn strength from eqn. (6), (7) and (8). Experimental polypropylene deformation at break and predicted strength were used.

The values of ultimate characteristics for blended cotton/polyester yarns, 100% cotton and 100% polyester yarns are presented in the tab.2. These yarns were spun from the same material by using of the ring technology. The experimental data and classical or optimal estimators for the case of cotton / polyester yarns are given on the fig. 3 and for the cotton / polypropylene yarns on the fig. 4.

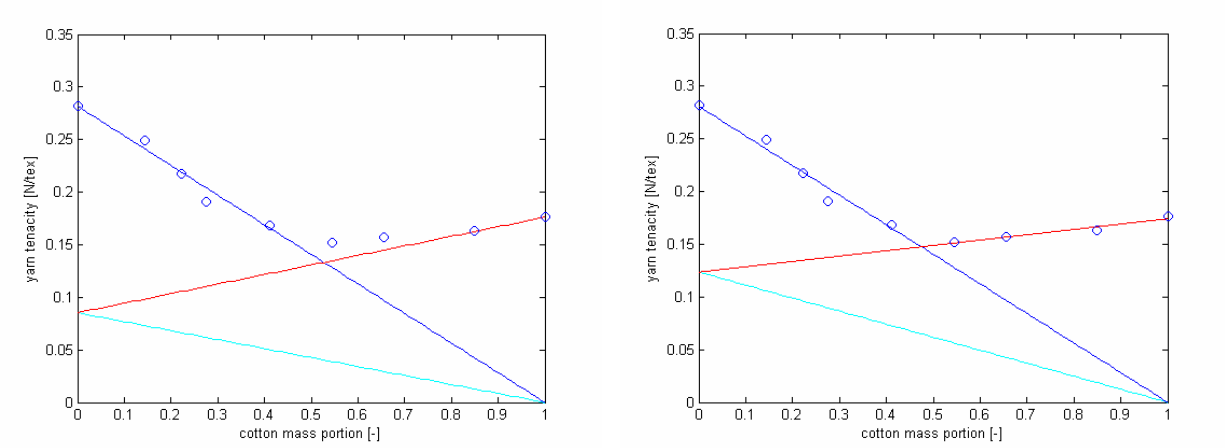
Table2 Ultimate characteristics for blended cotton/polyester yarns

Yarn n.	Raw material	Cotton mix. Ratio [-]	Fineness [tex]	Twist [m^{-1}]	Strength real [N/tex]	Elongation real [%]	Strength class. [N/tex]	Strength opt. [N/tex]	Strength pred. [N/tex]
1	cotton	1	10,1	945	0,1761	4,0	0,1761	0,1741	0,1799
2	PET/co	0,85	10,3	930	0,1633	4,2	0,1624	0,1665	0,1728
3	PET/co	0,66	10,6	953	0,1571	4,5	0,1448	0,1567	0,1637
4	PET/co	0,54	11,1	927	0,1518	5,4	0,1348	0,1511	0,1585
5	PET/co	0,41	11,1	958	0,1681	7,7	0,1659	0,1654	0,1598
6	PET/co	0,28	11,0	971	0,1906	9,9	0,2039	0,2033	0,1964
7	PET/co	0,22	12,0	955	0,2170	11,4	0,2187	0,2180	0,2107
8	PET/co	0,14	12,4	955	0,2492	12,3	0,2411	0,2404	0,2322
9	PET	0	12,9	948	0,2816	13,3	0,2816	0,2807	0,2712

Strength deviations δ_i (classical, optimal and predicted) are defined by the equation

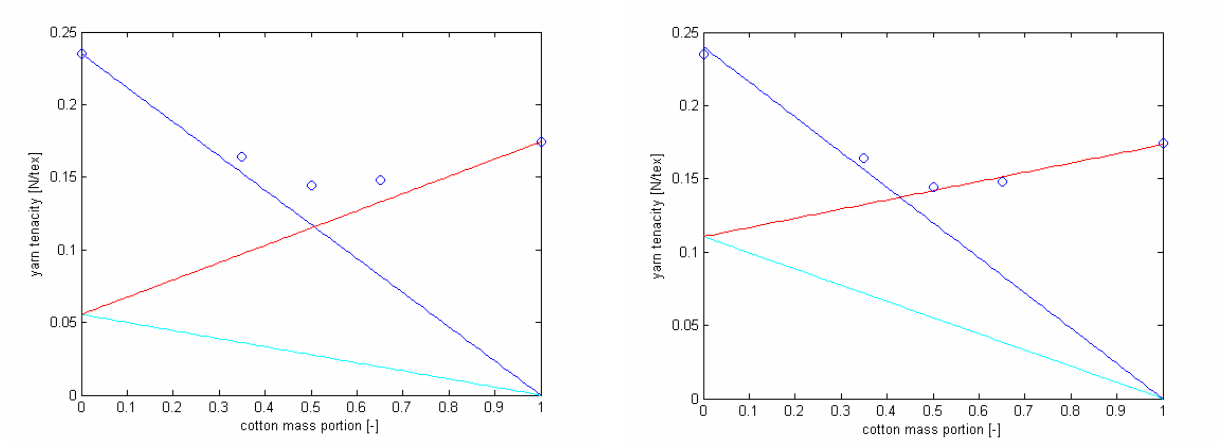
$$\delta_i = 100\left(\sigma_i - \sigma_r\right)/\sigma_r \qquad (20)$$

where σ_r is measured strength and σ_i is classical, optimal or predicted strength. These deviations for set of polyester / cotton yarns with various mixing ratios are shown on the fig.5.



a) classical b) optimal

Fig.3 Linear mixing rule for polyester/cotton yarns (12 tex)



a) classical b) optimal

Fig.4 Linear mixing rule for polypropylene/cotton yarns (20 tex)

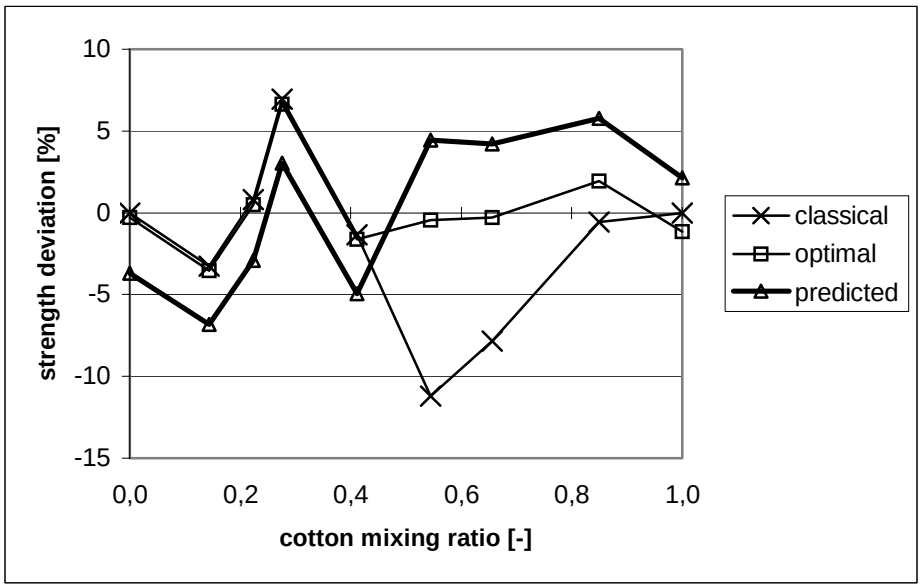


Fig.5 Strength deviation for set of polyester/cotton yarns (12 tex)

Discussion

It is visible that the optimal estimator computed from a set of experimental blended yarns is systematically lower than the classical estimator (see tab.1, fig.3 and 4). In fig.5, it is shown, that for the optimal estimator, the lowest is strength deviation. The predicted strength deviation is dependent on the prediction quality of pure cotton and pure polyester yarn. In this case, the predicted critical mixing ratio G_1 is surprisingly better that classical one. Prediction of cotton yarn strength by using eqn. (14) is satisfactory, but the problem is prediction of polyester and polypropylene yarn strength. Stress-strain curves are non linear and the parameter M is not sufficiently precise. The utilization of the Weibull distribution for prediction of bundle strength is then suitable. The main problem is prediction of deformation at break, too.

Conclusion

The prediction of the critical mixing ratio based on the Hamburger, is critically dependent on the accuracy of single components strength. For the case when the experimental strengths for set of yarns are at disposal the regression model described by eqn. (4) and (5) can be used.

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