INTERPRETATIONS OF SINGLE COTTON FIBER TENSILE PROPERTIES

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Abstract

The frequency distributions of breaking elongation, breaking strength and toughness of the cotton fibers are positively skewed and also have longer tails than normal distributions. Gamma distribution provides the best fit for the breaking elongation of the three cotton fibers. Weibull distribution provides the best fit for the breaking strength and toughness of cotton I and III, Beta distribution also provides the close fit for the breaking strength and toughness. However, only Gamma distribution fits the breaking strength and toughness of cotton II fibers, whereas neither Weibull nor Beta distribution can fit it.

Introduction

Fibers are the most elementary constituents in yarn, fabric, and composite structures. Their tensile properties are thus among the most important physical parameters from which the properties of their assemblies are determined. Measurements of single fiber tensile properties are time-consuming and impractical for large scale and industrial applications. For cotton fibers, methods involving measurements of fiber bundles have been developed. The tensile properties of single cotton fibers are then estimated from established correlation between the bundle fibers and single fibers.

For basic understanding of fiber tensile properties, direct measurements of single fibers are crucial. This is due to the scattering nature of single fiber tensile properties which are described not only by arithmetic mean values, but also by statistical distribution functions. The frequency distributions of single cotton fiber strength have been analyzed. Peirce early postulated that the strength of a single fiber is that of the weakest element along its length, which is then named as the weakest link theorem [Peirce, 1926]. This theorem was then applied to deduce a Weibull distribution, which had been demonstrated to fit the strength of cotton [Weibull, 1951, Cui, 1994]. Although Weibull distributions were generally accepted, Beta distributions were also claimed for interpreting the frequency distributions of the breaking force and tenacity as well as the ratio of cell wall thickness to ribbon width of cotton fibers [Somashekar et al, 1977; Patel et al, 1979]. For breaking elongation, either a normal distribution [Peirce, 1926, Cui, 1994] or a Rayleigh distribution [Frydrych, 1995] has been reported.

The presentence on the distributions of cotton breaking strength and elongation remains varying and requires further verification. Furthermore, little is known about the distributions of cotton fiber toughness. The aim of this work is to analyze the distributions of breaking strength, breaking elongation and toughness of single cotton fibers with more in-depth statistical analysis.

Experimental

Materials

Three types of cotton fibers were measured in this work. The three cottons were a micronaire standard (Cotton Inc.), a Mantis calibration standard (Zellweger Usters, Inc.), and fibers from a carded sliver (Nisshinbo Inc.). For ease of designation, these three cotton fibers are referred as Cotton I, II, III, respectively. All samples were conditioned at 70°F and 65% relative humidity for at least 48 hours prior to the measurements.

Single Fiber Tensile Measurement

A Mantis single fiber tensile tester (Zellweger Usters, Inc.) was used in this work to accumulate a large number of data [Hebert et al, 1995]. Tensile measurements are performed at a 3.2 mm gauge length using an internal pressure transducer at a 1.0 mm/s strain rate. In addition, fiber diameters or ribbon widths are determined by an optical sensor. All measurements were performed at 70°F and 65% relative humidity. For the cotton fibers, the coefficient of variation (*cv*) is in the range of 30-55%, thus the minimum sample size was found to be 300 [Warrier et al, 1982]. The tested sample sizes in this work were 997, 784, 497 for cotton I, II, III, respectively.

Results and Discussion

Table 1 summarized the extreme values, mean values, standard deviations, and coefficient of variations of the breaking elongation, breaking force and toughness of the three cotton fibers. The mean elongation value and standard deviation of cotton II fibers are the lowest whereas those of the cotton I are the highest. The coefficient of variation (*cv*), however, is the best parameter for comparing the variances among different typies of fibers. The cotton III fibers are found to have the highest *cv* value whereas the cotton I fibers have the lowest *cv* value.

For the breaking force, the mean value and standard deviation of cotton II are the highest. Cotton II fibers are considered more brittle since it has highest breaking force and lowest elongation in comparison to the other two cotton fibers. The breaking force of cotton II has the lowest *cv* value whereas that of cotton I has the highest *cv* value. A recent study has indicated significance of the standard deviation of the single fiber breaking elongation on the

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strength of fiber bundles [Fryer et al, 1996]. If it is indeed the case, fiber bundles composed of cotton II fibers may yield higher strength.

Cotton I fibers exhibit the highest mean toughness value and standard deviation whereas the cotton II fibers demonstrate the lowest mean values and standard deviations. Among the three cotton fibers, the cotton II fibers have the lowest *cv* values not only for the breaking force but also for the toughness. Cotton III has the highest toughness *cv* value.

Further analysis requires to test whether the frequency distributions obey normal distributions. The statistical testing involves the measurements for symmetry and long-tails. They are represented by skewness (g_1) and kurtosis (g_2), respectively:

$$g_1 = \Sigma(x_i - \bar{x})^3/(Ns^3) \tag{1}$$

$$g_2 = \Sigma(x_i - \bar{x})^4/(Ns^4) - 3 \tag{2}$$

The standard errors of g_1 and g_2 for a normal distribution are $(6/N)^{1/2}$ and $(24/N)^{1/2}$, respectively, depending on the number of measurements.

Table 2 summarizes the skewness (g_1) and kurtosis (g_2) of the obtained data. The frequency distributions of breaking elongation, breaking force and toughness are all positively skewed. All the calculated skewness values are larger than the standard errors, indicating asymmetrical distributions. The positive skewness values of these distributions also indicate longer tails on the right side of the distributions. The longer right tails have the physical meaning of larger amounts of cotton fibers possessing lower elongation, breaking force and toughness than their mean values.

The fact that larger amounts of fibers in these three cotton populations possess lower elongation, breaking force and toughness than the mean values has significant implications in the performance of cotton assemblies, such as bundles, yarns, and fabrics. The sources of these less extensible fibers or the weaker fibers are not clear. They may have to do with the varying development of the fibers, fiber processing, and blending. It should be noted that cotton II and III are slivers which have been ginned and carded or combed. The ginning, carding and other fiber processing may alter the fiber properties. Also, cotton I is likely the blends of several constituents. The elongation distribution of the blend has been reported not to be normally distributed, despite its constituent distributions [Fryer, 1996].

The kurtosis values of the breaking elongation, breaking force and toughness distributions of the three cotton fibers are also found to be significantly higher than the standard errors of normal distributions, indicating that these distributions have longer tails than normal distributions. Hence, these distributions should by no means be considered as normal distributions. The asymmetrical distributions shown by these three cotton populations do not support Cui et al 's claim that the cotton elongation distributions obey normal distributions.

For asymmetrical distributions, the statistical asymmetrical distributions, such as Weibull, Log-normal, Gamma, Gumbel, and Beta were employed for curve resolutions (Table 3). The corresponding parameters of each function were calculated by the numerical method of maximum likelihood estimation (MLE) using BMDP statistical computer programs [Dixon et al, 1985]. For Gamma and Beta distributions, the unknown parameters were estimated by the moment methods, namely, the sample mean value and the variance being set equal to the corresponding expressions for the population.

Figure 1 shows that all of the elongation frequency distributions are asymmetrical and have long tails than normal distributions. Our preliminary analysis showed that the Log-normal and Gumbel distributions do not fit the experimental data since they have sharper peaks with longer tails on the right sides. Therefore the curve-fittings were only performed by Weibull, Gamma, and Beta distributions (Figure 1). The Weibull and Beta distributions also do not fit the elongation data near the top or in the peak regions. This means that our analyses of these breaking elongation distributions do not support the Rayleigh distribution [Frydrych, 1995], since the Rayleigh distribution is only a special case of Weibull distribution when the shape parameter *m* is equal to two.

The Gamma distribution seems to be the best fit for the elongations of all the three cotton populations. The position parameter *c* has been set to be the minimum observation of each sample. For our cotton fibers, it is found that the shape parameter *m* is in the range from 7.5 to 9.0 whereas the scale parameter *a* is in the range from 0.7 to 0.9 (Table 4). The chi-square tests were employed for evaluating the goodness of fit. Under 95% confidencial level, the chi-square results confirm that the breaking elongations follow the Gamma distributions.

The breaking strength frequency distributions of all three cotton fibers also appear to be asymmetrical and have longer tails than normal distributions (Figure 2). The corresponding parameters for the employed fitting distributions are summarized in Table 5. The Weibull distribution provides the best fits to the actual data for cotton I and cotton III, while Beta distribution provides the second best fit for the same fibers. Under 95% confidencial level, the chi-square results also show that the Weibull and Beta distributions fit the distributions of cotton fiber strength with Weibull being the best. Thus it is not surprised that Weibull and Beta distributions were all claimed for the cotton fiber strength in literature. The difference is that Beta distribution tends to have broader peak near half peak height region and shorter tails on both ends. For our cotton fibers, it has shown that the scale parameter *a* of a Weibull distribution is in the range of 83.6-156.7 while the shape parameter *m* is in the range of 2.6-2.8.

However, for that distribution with sharp peak such as cotton II, it has been found that a Weibull distribution does not fit the distribution well. A Weibull distribution often is broad at the peak maximum amplitude. Gamma distribution provides the best fit in this case. Therefore although Weibull distribution is more applicable to cotton I and III, Gamma distribution are suitable for cotton II.

Figure 3 shows the toughness frequency distributions of these three cotton fibers with the curve-fittings. The corresponding parameter estimations are summarized in Table 6. Again, Weibull distributions are found to be the best fit for the cotton I and II fibers. For these cotton fibers, the shape parameter *m* is in the range from 1.87 to 2.06 whereas the scale parameter *a* is in the range from 79.11 to 118.06. Although the shape parameters of the Weibull distributions are found to close to 2, the toughness of cotton fibers should not be considered as from Rayleigh distribution population. Furthermore, Gamma distribution again is found to be the best fit for the toughness of cotton II.

Summary

Single cotton fiber breaking elongation, strength and toughness have been measured and their distributions analyzed. The breaking elongation, strength and toughness distributions of the three cotton fibers are all positively skewed and have longer tails than normal distributions. Therefore, these distributions can not be considered as normal distributions. For the elongations, Gamma distributions seem to provide the best-fit when comparisons are made with several other statistical distributions (Beta, Weibull, Lognormal). For the breaking strength and toughness, Weibull distributions usually provides the best-fit while the Beta distributions provide the second best fit. For the breaking strength and toughness of cotton II, however, Gamma distribution provides the best fit.

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Table 1. The measured data of single fiber tensile properties

Sample	x_{min}	x_{max}	$\bar{x}$	s	cv
Elongation					
Cotton I	0.1	29.0	12.6	4.2	33.2
Cotton II	2.6	23.7	9.9	3.3	33.5
Cotton III	1.6	26.2	10.4	3.8	36.3
Breaking force					
Cotton I	0.8	12.8	5.0	2.1	42.9
Cotton II	1.0	18.9	5.8	2.3	38.8
Cotton III	0.9	15.0	5.5	2.2	39.2
Toughness					
Cotton I	0.0	29.5	9.8	5.2	52.9
Cotton II	0.5	30.6	9.0	4.6	51.1
Cotton III	0.3	41.4	9.2	5.2	56.1

Table 2. The skewness (g_1) and kurtosis (g_2) of the obtained data

Sample	g_1	g_2	g_{1c}	g_{2c}
Elongation				
Cotton I	0.53	0.44	0.08	0.16
Cotton II	0.70	0.79	0.09	0.20
Cotton III	0.90	1.60	0.11	0.22
Breaking force				
Cotton I	0.53	0.02	0.08	0.16
Cotton II	0.95	2.13	0.09	0.20
Cotton III	0.35	0.14	0.11	0.22
Toughness				
Cotton I	0.72	0.38	0.08	0.16
Cotton II	0.86	1.04	0.09	0.20
Cotton III	1.17	3.23	0.11	0.22

Table 3. Employed asymmetric standard statistical functions and their expected values and variances.

Function	pdf
Log-normal $Ln(c,m^2)$	$p_{Ln}(x)=\begin{cases}\frac{1}{\sqrt{2\pi}mx}e^{-\frac{(\ln x-c)^2}{2m^2}},\ldots x>0\\ 0,\ldots\ldots\ldots\ldots x\leq\end{cases}$ $-\infty<c<\infty,\quad m>0$
Weibull $W(c,\alpha,m)$	$p_w(x)=\begin{cases}\frac{m}{\alpha}(x-c)^{m-1}e^{-\frac{(x-c)^m}{\alpha}},\ldots x\geq c\\ 0,\ldots\ldots\ldots x<c\end{cases}$ $m>0,\quad \alpha>0$
Gamma $\Gamma(c,\alpha,m)$	$p_\Gamma(x)=\begin{cases}\frac{\alpha^m}{\Gamma(m)}(x-c)^{m-1}e^{-\alpha(x-c)},\ldots x>c\\ 0.,\ldots\ldots\ldots\ldots x\leq c\end{cases}$ $\alpha>0,\quad m>0$
Beta $\beta(m,n)$	$p_\beta(x)=\begin{cases}\frac{(x-c)^{m-1}\times(c+\alpha-x)^{n-1}}{\alpha^{m+n-1}\times B(m,n)},\ldots c\leq x\leq(c+\alpha)\\ 0..,\ldots\ldots x<c,\ldots.x>(c+\alpha)\end{cases}$ $m>0,\quad n>0$

Table 4. Estimated parameters for the breaking elongation data.

Sample	c	α	m	n
Log-normal				
Cotton I	2.5		0.38	
Cotton II	2.2		0.35	
Cotton III	2.3		0.37	
Weibull				
Cotton I	2.0	813.6	2.71	
Cotton II	2.3	176.5	2.41	
Cotton III	2.8	100.3	2.14	
Gamma				
Cotton I	0.1	0.72	8.99	
Cotton II	0.3	0.90	8.90	
Cotton III	0.2	0.74	7.62	
Beta				
Cotton I	0.1	28.9	4.65	6.11
Cotton II	0.3	21.1	2.84	5.31
Cotton III	0.2	24.6	3.18	5.71

Table 5. Estimated parameters for the breaking force data

Sample	c	α	m	n
Log-normal				
Cotton I	1.5		0.46	
Cotton II	1.7		0.40	
Cotton III	1.6		0.45	
Weibull				
Cotton I	0.0	83.6	2.56	
Cotton II	0.0	156.7	2.70	
Cotton III	0.0	152.0	2.75	
Gamma				
Cotton I	0.8	1.14	5.69	
Cotton II	1.0	1.15	6.65	
Cotton III	0.8	1.18	6.52	
Beta				
Cotton I	0.8	12.0	2.27	4.17
Cotton II	1.0	17.9	3.03	8.31
Cotton III	0.8	14.1	2.77	5.63

Table 6. Estimated parameters for the toughness data.

Sample	c	α	m	n
Log-normal				
Cotton I	2.2		0.63	
Cotton II	2.1		0.59	
Cotton III	2.0		0.65	
Weibull				
Cotton I	0.0	118.5	1.99	
Cotton II	0.0	118.6	2.06	
Cotton III	0.0	79.1	1.87	
Gamma				
Cotton I	0.4	0.37	3.59	
Cotton II	0.5	0.43	3.83	
Cotton III	0.3	0.35	3.18	
Beta				
Cotton I	0.4	29.1	1.92	4.02
Cotton II	0.5	30.1	2.17	5.53
Cotton III	0.3	41.1	2.13	7.70

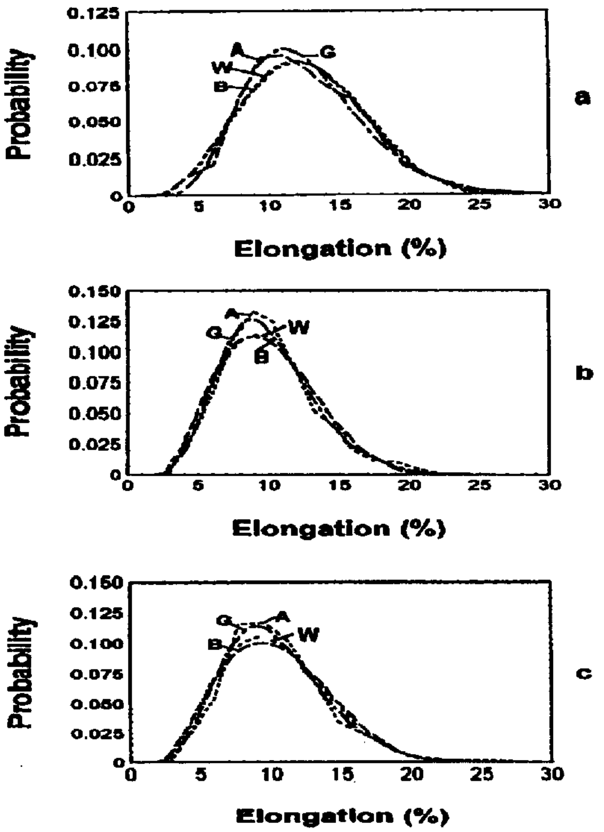

Figure 1. Experimental and the resolved elongation distributions with various standard statistical distribution functions for the cotton fibers (**A**--actual data, **W**--Weibull, **G**--Gamma, **B**--Beta): a. cotton I, b. cotton II, c. cotton III.

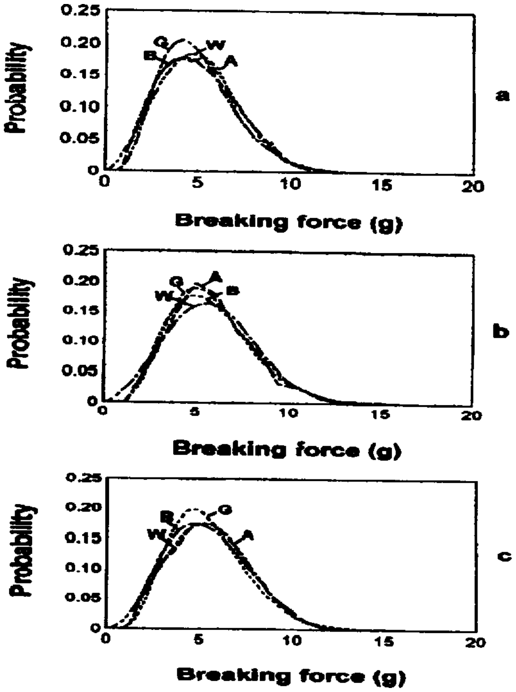

Figure 2. Experimental and the resolved breaking strength distributions with various standard statistical distribution functions for the cotton fibers (**A**--actual data, **W**--Weibull, **G**--Gamma, **B**--Beta): a. cotton I, b. cotton II, c. cotton III.

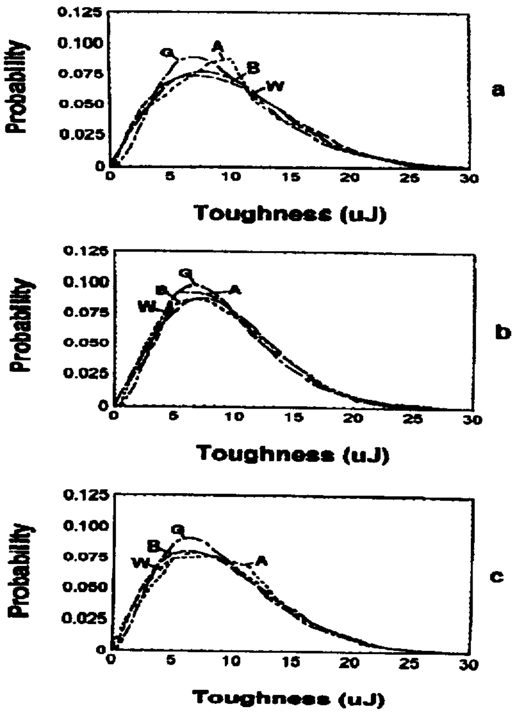

Figure 3. Experimental and the resolved toughness distributions with various standard statistical distribution functions for the cotton fibers (**A**--actual data, **W**--Weibull, **G**--Gamma, **B**--Beta): a. cotton I, b. cotton II, c. cotton III.